Outage Achievable Rate Analysis for the Non Orthogonal Multiple Access Multiple Relay Channel

Abdulaziz Mohamad*, Raphaël Visoz*, Antoine O. Berthet†
*Orange Labs, Issy les Moulineaux, France
†SUPELEC, Department of Telecommunications, France
{mohamad.abdulaziz; raphael.visoz}@orange-ftgroup.com; {antoine.berthet}@supelec.fr

***Abstract*—In this paper, we derive the individual and common symmetric ϵ-outage achievable rate for the relay assisted cooperative communication scheme, coined Non Orthogonal Multiple Access Multiple Relay Channel (NOMAMRC), defined as follows: (1) The sources are independent and want to communicate with a single destination with the help of multiple relays; (2) Each relay is half-duplex and apply a Selective Decode and Forward (SDF) strategy, i.e., it forwards only a deterministic function of the error-free decoded messages; (3) The sources are allowed to transmit simultaneously during both the listening and transmission phases of the relays. In this paper, the links are independent. No channel state information at transmitter is available, each link follows a slow Rayleigh fading distribution.**

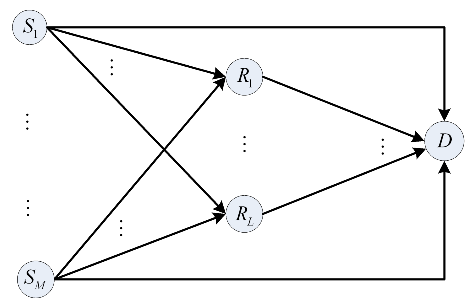

Fig. 1. A Multiple-Access Multiple-Relay channel

I. Introduction

The multiple access channel (MAC) consists of M independent users (sources) accessing the same radio resource to communicate with a common destination. The MAC capacity region is well known, see, e.g., [1]. However, the use of relays to enlarge the MAC capacity region has motivated many research efforts during the past decade. In this paper, we investigate, from an information theoretic perspective, an M-user MAC assisted by L relay cooperation scheme (see Fig.1) defined as follows. Each relay is half-duplex and implements the Selective Decode and Forward (SDF) strategy, i.e., the relay forwards a deterministic function of the source's messages that it can decode error-free. As a result, the transmission time of the messages of the sources is divided into two phases. In the first phase, the M sources simultaneously broadcast the first part of their codeword (which is itself a codeword) creating a first M-user MAC at the destination and at each relay. In the second phase, both the sources and the $L_a \leq L$ relays, that were able to decode at least one source message error free, are active which creates a second $(M + L_a)$-user MAC at the destination. More specifically, the sources continue to transmit the second part of their codeword while the relays transmit some joint redundancy on the sources messages that were decoded error free. We call this relay assisted cooperative communication scheme Non-Orthogonal Multiple Access Multiple Relay Channel (NOMAMRC). In this paper, we assume that no channel state information is available at transmitter and that the links are independent, and follow a slow Rayleigh fading distribution. As a result, information theoretic outage events always occur in our system and their analysis is the subject of this paper. Sending a deterministic function of the received messages at layer 3 of the OSI model instead of routing each received message separately is optimal in the sense that it can achieve the min cut max flow capacity of the underlying network [2] and is coined network coding. Here, we consider wireless network coding where the network coding is performed at the physical layer. From this viewpoint, the proposed SDF strategy cannot be proven to be optimal. We restrict our outage analysis to this particular relaying strategy because (1) It breaks down the MAMRC into parallel MACs whose individual outage regions are perfectly known; (2) It prevents the error propagation from the relay to the destination and decreases the individual outage probability associated to each source (in most practical tested scenarios), (3) It reduces the energy consumption at the relays and limits the interference within the network (the relay when it cooperates is always helpful). We are well aware that other competing strategies exist in the literature, see, e.g., [3], but theirs comparisons with SDF exceed the scope of this paper and is left for further studies. We deliberately include the SDF strategy into the NOMAMRC. Based on this definition, we are able to determine individual and common symmetric achievable rate region conditional to the channel states of the NOMAMRC. However, if we relax the constraint of the SDF strategy it is well known that even the capacity of the simple relay channel is unknown [4]. By building on the M-user MAC outage analysis, see, e.g. [5], [6], this paper formulates the symmetric individual and common NOMAMRC ϵ-outage achievable rate conditional to the chosen input distributions. The symmetric individual ϵ-outage achievable rate is defined as the highest transmission rate of each source such that the probability of any source to be in outage is less or equal to

Part of this work has been performed in the framework of the FP7 project ICT-317669 METIS, which is partly funded by the European Union. The authors would like to acknowledge the contributions of their colleagues in METIS, although the views expressed are those of the authors and do not necessarily represent the project.

ϵ. The symmetric common ϵ-outage achievable rate is defined as the highest transmission rate of each source such that the probability of a common outage event, which is defined as the event of having at least one source in outage, is less or equal to ϵ. This paper complements [7] which focuses on the NOMARC outage analysis for two sources and one relay, by providing a general outage analysis framework for any number of sources and relays. Interestingly, [7] shows that distributed turbo-coding with Joint Network Channel Decoding (JNCD) at the destination is able to get close to the NOMARC ϵ-outage achievable rate for QPSK entries. Finally, Monte Carlo simulations that compare the symmetric individual and common ϵ-outage achievable rate of NOMAMRC with respect to no cooperation at all, are presented and discussed.

In Section II, we introduce the System Model of NOMAMRC. Section III is devoted to outage analysis. Numerical results will be presented in Section IV. Some conclusions are drawn in Section V.

Notation Calligraphic upper case letters are used to denote integer finite sets of the form $\mathcal{S} = \{s_1, \ldots, s_{|\mathcal{S}|}\}$ where $|\mathcal{S}|$ is the cardinality of the set $\mathcal{S}$. The empty set is denoted by $\emptyset$. Let $\mathbf{x}_\mathcal{S}$ denote the vector $[x_{s_1}, \ldots, x_{s_{|\mathcal{S}|}}]^\top$ where $^\top$ is the transpose operator. Similarly, we define the block vector $\underline{\mathbf{x}}_\mathcal{S}$ as $[\mathbf{x}_{s_1}^\top, \ldots, \mathbf{x}_{s_{|\mathcal{S}|}}^\top]^\top$. $x \sim \mathcal{CN}(\mu, \sigma^2)$ means that x is a circularly-symmetric complex Gaussian random variable with mean μ and covariance σ^2. Given a condition C we define $\mathbf{1}_{\{C\}}$ as an indicator function, i.e., $\mathbf{1}_{\{C\}} = 1$ if C is true and $\mathbf{1}_{\{C\}} = 0$ otherwise.

II. NOMAMRC System Model

Let us consider a set of statistically independent sources $\mathcal{S} = \{s_1, \ldots, s_M\}$, each source $s \in \mathcal{S}$ wants to communicate its message $\mathbf{u}_s \in \mathbb{F}_2^K$ of K information bits to a common destination d with the help of a set of relays $\mathcal{R} = \{r_1, \ldots, r_L\}$. The sources, the relays and the destination are equipped with a single transmit and receive antenna and all the channel coefficients are slow fading and independent. Let N be the total number of available complex dimensions to be shared between the relays and the sources. These available channel uses are divided into two successive time slots corresponding to the listening phase of the relays, say $N_1 = \alpha N$ channel uses, and to the transmission phase of the relays, say $N_2 = \overline{\alpha} N$, where $\overline{\alpha} = 1 - \alpha$, with $\alpha \in]0, 1]$. During the first phase, each source $s \in \mathcal{S}$ transmit the first part of its modulated codeword denoted by $\mathbf{x}_s^{(1)} \in \mathbb{C}^{N_1}$ (which is itself a codeword). The associated received signals at the relay $r \in \mathcal{R}$ and the destination d can be written as

$$y_{r,k}^{(1)} = \sum_{s \in \mathcal{S}} h_{sr} x_{s,k}^{(1)} + n_{r,k}^{(1)}, \tag{1}$$

and

$$y_{d,k}^{(1)} = \sum_{s \in \mathcal{S}} h_{sd} x_{s,k}^{(1)} + n_{d,k}^{(1)} \tag{2}$$

where $k = 1, \ldots, N_1$, respectively. The channel fading coefficients h_{sr} and h_{sd} follow the pdf $\mathcal{CN}(0, \gamma_{sr})$ and $\mathcal{CN}(0, \gamma_{sd})$, respectively. The AWGN samples $n_{r,k}^{(1)}$ and $n_{d,k}^{(1)}$ are i.i.d and follow the pdf $\mathcal{CN}(0, 1)$. During the second phase, each source s continue to transmit the second part of their modulated codeword denoted by $\mathbf{x}_s^{(2)} \in \mathbb{C}^{N_2}$. Thus, each source s has encoded its K information bits into a hierarchical codeword $\mathbf{x}_s = [\mathbf{x}_s^{(1)\top}, \mathbf{x}_s^{(2)\top}]^\top$ transmitted over the N available channel uses, it results an overall system spectral efficiency of MR bits per channel use (bits/c.u.) where $R = K/N$ is the individual rate of each source. Let $\mathcal{S}_r \subseteq \mathcal{S}$ denote the maximum set of sources' messages that the relay r can decode error free. If the set $\mathcal{S}_r$ is empty then r remains silent in this phase. Otherwise, the relay r transmits a modulated sequence $\mathbf{x}_r^{(2)} \in \mathbb{C}^{N_2}$. The sequence $\mathbf{x}_r^{(2)}$ is chosen such that $[\underline{\mathbf{x}}_{\mathcal{S}_r}^\top, \mathbf{x}_r^{(2)\top}]^\top$ is a (modulated) codeword on the messages $\underline{\mathbf{u}}_{\mathcal{S}_r}$. The received signal at the destination can be written as

$$y_{d,k}^{(2)} = \sum_{s \in \mathcal{S}} h_{sd} x_{s,k}^{(2)} + \sum_{r \in \mathcal{R}_a} h_{rd} x_{r,k}^{(2)} + n_{d,k}^{(2)} \tag{3}$$

where $k = 1, 2, \ldots, N_2$ and $\mathcal{R}_a$ denotes the set of active relays. The channel fading coefficient h_{rd} is independent from the other ones and follows the pdf $\mathcal{CN}(0, \gamma_{rd})$ while the additive AWGN noise samples $n_{d,k}^{(2)}$ are i.i.d and follow the pdf $\mathcal{CN}(0, 1)$. We further assume that the transmitted symbols' power (per complex dimension) from the sources and the relays are normalized to unity.

III. Information-Theoretic analysis

In this Section, we assume classically that $N \to \infty$ and that all the transmitted sequences are i.i.d such that the AEP holds. The NOMAMRC breaks down into one MAC at each relay and one MAC at the destination corresponding to the first phase, and one MAC at the destination corresponding to the second phase thanks to the SDF relaying function. Thus, its outage region is perfectly known conditional on the NOMAMRC channel states $\mathbf{h} = [\mathbf{h}_{\mathcal{S}r_1}^\top, \ldots, \mathbf{h}_{\mathcal{S}r_L}^\top, \mathbf{h}_{\mathcal{S}d}^\top, \mathbf{h}_{\mathcal{R}d}^\top]^\top$ and the inputs distributions $p(\mathbf{x}_\mathcal{S}) = \prod_{s \in \mathcal{S}} p(x_s)$ and $p(\mathbf{x}_{\mathcal{R}_a}^{(2)}) = \prod_{r \in \mathcal{R}_a} p(x_r^{(2)})$ corresponding to the underlying joint pdfs of the source transmitted sequences and the active relay ones, respectively (discrete entry distributions are considered as dirac comb pdfs). Note that we further assume that $p(\mathbf{x}_{\mathcal{R}_a}^{(2)}, \mathbf{x}_\mathcal{S}) = p(\mathbf{x}_{\mathcal{R}_a}^{(2)}) p(\mathbf{x}_\mathcal{S})$. This independence property is the core principle of Joint Network Channel Coding at the relays.

A. Slow fading MAC Outage analysis

When every relays can not decode any source message error free, the NOMARC becomes a simple MAC at the destination (no-cooperation case). As a result, the received signal at the destination can be written as

$$y_{d,k} = \sum_{s \in \mathcal{S}} h_{sd} x_{s,k} + n_{d,k} \tag{4}$$

for $k = 1, 2, \ldots, N$. The symmetric achievable rate region of a $|\mathcal{S}|$-user MAC [1], [8] is the complement of the closure of the convex hull of the rate vectors satisfying

$$R_\mathcal{U} \leq I(\mathbf{x}_\mathcal{U}; y_d | \mathbf{x}_{\mathcal{U}^c}, \mathbf{h}_{\mathcal{S}d}) \quad \text{for all} \quad \mathcal{U} \subseteq \mathcal{S} \tag{5}$$

where $\mathcal{U}^c = \mathcal{S} \backslash \mathcal{U}$, $R_\mathcal{U} = |\mathcal{U}| R$ and given the input distribution $\prod_{s \in \mathcal{U}} p(x_s)$. For the sake of notation simplicity, we remove the channel state from the outage event definitions and mutual information expressions in the following. Let $\mathcal{O}_{d,s}$ denote the symmetric individual outage event of source s, and $\mathcal{E}_{d,\mathcal{S}}$ denote the symmetric common outage event at the destination d of the $|\mathcal{S}|$-user MAC. Using (5) this event could be expressed as

$$\mathcal{E}_{d,\mathcal{S}} = \{R_{\mathcal{U}} > I(\mathbf{x}_{\mathcal{U}}; y_d|\mathbf{x}_{\mathcal{U}^c}) \quad \text{for some} \quad \mathcal{U} \subseteq \mathcal{S}\} \tag{6}$$

equivalently

$$\mathcal{E}_{d,\mathcal{S}} = \bigcup_{\mathcal{U}\subseteq\mathcal{S}} \mathcal{F}_{d,\mathcal{S}}(\mathcal{U}) \tag{7}$$

where $\mathcal{F}_{d,\mathcal{S}}(\mathcal{U})$ is defined as the outage event of sources $\mathcal{U}$, the messages of $\mathcal{U}^c = \mathcal{S}\backslash\mathcal{U}$ being perfectly known. This event can be expressed as

$$\mathcal{F}_{d,\mathcal{S}}(\mathcal{U}) = \{R_{\mathcal{U}} > I(\mathbf{x}_{\mathcal{U}}; y_d|\mathbf{x}_{\mathcal{U}^c})\} \tag{8}$$

When any $\mathcal{F}_{d,\mathcal{S}}(\mathcal{U})$ holds, the destination d can not decode all the messages of $\mathcal{U}$ knowing perfectly the messages of $\mathcal{U}^c$. In this case, a symmetric common outage of sources $\mathcal{S}$ is declared. The fact that $\mathcal{E}_{d,\mathcal{S}}$ holds does not mean that the destination can not decode error free the messages of a subset of $\mathcal{S}$. Excluding $\mathcal{S}$ itself and the empty subset, we can define $2^M - 2$ reduced MACs as follows

Definition (1) *A $|\mathcal{I}^c|$-user reduced MAC is a MAC with a subset of sources $\mathcal{I}^c$ of the original MAC, considering the complement of this subset $\mathcal{I} = \mathcal{S}\backslash\mathcal{I}^c$ as interference.*

Definition (2) *An expanded MAC of a $|\mathcal{I}^c|$-user reduced MAC is a MAC that contains at least the $\mathcal{I}^c$ original sources plus one.*

Let $\mathcal{E}_{d,\mathcal{I}^c}$ denote the common outage event of the $|\mathcal{I}^c|$-user reduced MAC. We can express this event by

$$\mathcal{E}_{d,\mathcal{I}^c} = \bigcup_{\mathcal{U}\subseteq\mathcal{I}^c} \mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U}) \tag{9}$$

where

$$\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U}) = \{R_{\mathcal{U}} > I(\mathbf{x}_{\mathcal{U}}; y_d|\mathbf{x}_{\mathcal{U}_c})\} \tag{10}$$

defining $\mathcal{U}_c = \mathcal{I}^c \setminus \mathcal{U}$. This equation is very similar to (7). However, in $\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U})$ the set of sources $\mathcal{I}$ are considered as interference, i.e, only the sources belonging to $\mathcal{U}_c$ are supposed to be perfectly known.

Proposition (1) *The source s is in outage iff the $|\mathcal{S}|$-user MAC and all the reduced MAC containing s are in outage.*

$$\mathcal{O}_{d,s} = \bigcap_{\mathcal{I}\subset\mathcal{S}: s\in\mathcal{I}^c} \mathcal{E}_{d,\mathcal{I}^c} \tag{11}$$

$proof$: The sufficient part: if all the reduced MAC containing s are in outage then the message of s can not be decoded (error free) by any possible mean, thus, the source s is in outage. The necessary part: if the source s is in outage and one reduced MAC including this source is not in outage, it means that the destination can jointly decode the sources of this reduced MAC, as a result, the destination can decode the message of user s which contradicts the statement that s is in outage. ■

In some cases, i.e., at the relays, we are interested in finding $\mathcal{S}_d$ the maximum set of sources that the destination can decode.

Proposition (2) *The sufficient and necessary condition for a set of sources to be $\mathcal{S}_d$ is (i) the $|\mathcal{S}_d|$-user reduced MAC is not in outage and (ii) all the expanded MAC of this $|\mathcal{S}_d|$-user reduced MAC are in outage.*

$proof$: First, the fact that the $|\mathcal{S}_d|$-user reduced MAC is not in outage is a sufficient and necessary condition for the set $\mathcal{S}_d$ to be jointly decoded error free. In order to ensure that it is the maximum set, any other sub-set with higher cardinality cannot be decoded jointly error free. This is guaranteed by the second condition of the proposition. ■

Finally, the source s symmetric individual and common outage probability can be expressed as

$$P_{out,ind} = \int_{\mathbf{h}} \mathbf{1}_{\{\mathcal{O}_{d,s}(\mathbf{h})\}} p(\mathbf{h}) d(\mathbf{h}) \tag{12}$$

and

$$P_{out,com} = \int_{\mathbf{h}} \mathbf{1}_{\{\mathcal{E}_{d,\mathcal{S}}(\mathbf{h})\}} p(\mathbf{h}) d(\mathbf{h}), \tag{13}$$

respectively, where $\mathbf{h} = \mathbf{h}_{\mathcal{S}d}$. We define the symmetric individual and common ϵ-outage achievable rate by

$$R_\epsilon^{ind} = sup\{R : P_{out,ind} \leq \epsilon\} \tag{14}$$

and

$$R_\epsilon^{com} = sup\{R : P_{out,com} \leq \epsilon\}, \tag{15}$$

respectively.

B. General NOMAMRC Outage Analysis

1) Relay Outage Analysis: During the first transmission phase, we have an $|\mathcal{S}|$-user MAC at each relay $r \in \mathcal{R}$. Thus, the definition and analysis of the previous Section remain valid simply by replacing the index d by r and the event defined in (10) by

$$\mathcal{F}_{r,\mathcal{I}^c}(\mathcal{U}) = \{R_{\mathcal{U}} > \alpha I(\mathbf{x}_{\mathcal{U}}; y_r|\mathbf{x}_{\mathcal{U}_c})\} \tag{16}$$

where α comes from the fact that this $|\mathcal{S}|$-user MAC is active over αN channel use. Since each relay $r \in \mathcal{R}$ relies on the SDF strategy, we are interested in finding the maximum set of sources $\mathcal{S}_r$ with whose the relay r can cooperate with. The set $\mathcal{S}_r$ determine the cooperation mode of the relay r. As a result, we have 2^M cooperation modes for each relay and we have 2^{ML} cooperation modes considering the relays altogether.

2) Destination Outage Analysis: In this Section, we assume the best decoding scheme at the destination, i.e., it relies on joint network channel decoding (see, e.g., [9]). Since the source-to-destination and the source-and-relay-to-destination MACs associated with each successive transmission phase are non interfering, they can be viewed as one $|\mathcal{S}|$-user MAC made of two parallel MACs. As a result, the individual outage event of source s is given by (11) and the common outage event is given by (9). To compute $\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U})$, we need to partition the relays into three sets (i) if $r \in \mathcal{R}_I$ the relay signal x_r is interference, (ii) if $r \in \mathcal{R}_k$ the relay signal x_r is perfectly known, (iii) if $r \in \mathcal{R}_u$ the relay signal x_r and $\mathbf{x}_{\mathcal{U}}$ help to decode jointly error free the messages $\mathbf{u}_{\mathcal{U}}$. The definitions of the sets $\mathcal{R}_I$, $\mathcal{R}_k$, $\mathcal{R}_u$ are given below where $\mathcal{U}_c = \mathcal{I}^c \setminus \mathcal{U}$

Definition (3) *$\mathcal{R}_I = \{r \in \mathcal{R}_a : \mathcal{I} \neq \emptyset \wedge \mathcal{I} \subseteq \mathcal{S}_r\}$ the set of relays whose signals are interference in $\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U})$*

Definition (4) *$\mathcal{R}_k = \{r \in \mathcal{R}_a : \mathcal{S}_r \subseteq \mathcal{U}_c\}$ the set of relays whose signals are known without decoding in $\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U})$.*

Definition (5) *$\mathcal{R}_u = \mathcal{R}_a \backslash \{\mathcal{R}_k \cup \mathcal{R}_I\}$ the set of relays whose signals are to be jointly decoded with the sources belonging to $\mathcal{U}$ in $\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U})$*

Using the above partitioning, we can express $\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U})$ as

$$\begin{aligned}\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U}) = \{R_{\mathcal{U}} > \alpha I(\mathbf{x}_{\mathcal{U}}^{(1)}; y_d | \mathbf{x}_{\mathcal{U}_c}^{(1)}) + \\ \bar{\alpha} I(\mathbf{x}_{\mathcal{U}}^{(2)}, \mathbf{x}_{\mathcal{R}_u}^{(2)}; y_d | \mathbf{x}_{\mathcal{U}_c}^{(2)}, \mathbf{x}_{\mathcal{R}_k}^{(2)})\}.\end{aligned} \tag{17}$$

where $p(\mathbf{x}_{\mathcal{S}}^{(1)}) = p(\mathbf{x}_{\mathcal{S}}^{(2)}) = p(\mathbf{x}_{\mathcal{S}})$. Note that we can never claim that the Gaussian input distributions are the one that minimizes the NOMAMRC outage probability. Indeed, the individual outage computations at the relay involve interferences whose statistics depend on the input distributions. To illustrate (17), we consider the following example.

Example (1) *Consider a 3-user 3-relay NOMAMRC and take the following scenario for the relays cooperation $\mathcal{S}_{r_1} = \{s_1\}$, $\mathcal{S}_{r_2} = \{s_1, s_2\}$, and $\mathcal{S}_{r_3} = \{s_2, s_3\}$. Now let us consider, during our search to find if source s_2 is in outage or not, the event $\mathcal{F}_{d,\{s_1,s_2\}}(\{s_2\})$ witch is the outage event of source s_2 assuming that the signal s_1 is known and s_3 is interference. In this case, the signal of r_3 is interference, r_1 is known, and r_2 is a part of the codeword corresponding to source s_2 (conditional to the knowledge of the signal of s_1). So we can write*

$$\begin{aligned}\mathcal{F}_{d,\{s_1,s_2\}}(\{s_2\}) = \{R > \alpha I(x_{s_2}^{(1)}; y_d | x_{s_1}^{(1)}) + \\ \bar{\alpha} I(x_{s_2}^{(2)}, x_{r_2}^{(2)}; y_d | x_{s_1}^{(2)}, x_{r_1}^{(2)})\}\end{aligned}$$

□

For Gaussian input distributions the instantaneous mutual information expressions in $\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U})$ can be expressed as

$$I(\mathbf{x}_{\mathcal{U}}^{(1)}; y_d | \mathbf{x}_{\mathcal{U}_c}^{(1)}) = \log\left(1 + \frac{\sum_{s \in \mathcal{U}} |h_{sd}|^2}{1 + \sum_{s \in \mathcal{I}} |h_{sd}|^2}\right) \tag{18}$$

$$\begin{aligned}I(\mathbf{x}_{\mathcal{U}}^{(2)}, \mathbf{x}_{\mathcal{R}_u}^{(2)}; y_d | \mathbf{x}_{\mathcal{U}_c}^{(2)}, \mathbf{x}_{\mathcal{R}_k}^{(2)}) = \\ \log\left(1 + \frac{\sum_{s \in \mathcal{U}} |h_{sd}|^2 + \sum_{r \in \mathcal{R}_u} |h_{rd}|^2}{1 + \sum_{s \in \mathcal{I}} |h_{sd}|^2 + \sum_{r \in \mathcal{R}_I} |h_{rd}|^2}\right).\end{aligned} \tag{19}$$

IV. Numerical Results

In this Section, we consider only Gaussian i.i.d inputs, where the instantaneous mutual information are computed using (18) and (19). The symmetric ϵ-outage achievable rate is computed for $\epsilon = 0.01$. There are an infinity of SNR configurations γ_{rd}, γ_{sr}, γ_{sd}. By taking a few arbitrary configurations as examples, we want to illustrate in this simulation Section that the NOMAMRC always outperforms the M-MAC (no cooperation at all) even in the presence of noisy slow fading source-to-relay links.

In the first set of simulations, we choose $\alpha = 2/3$, $\gamma_{rd} = \gamma$ and $\gamma_{sr} = \gamma_{sd} = \alpha\gamma$. We consider this choice in order to validate our results with the ones of [7] obtained for NOMARC. Fig. 2, and 3 show the symmetric individual and common ϵ-outage achievable rate, respectively. The number of sources ranges from $M = 1$ to $M = 4$ while the number of relays ranges from $L = 0$ to $L = 3$. Clearly, for $L = 0$ we have the M-user MAC sub-case (including the point-to-point channel when $M = 1$). First of all, we observe that the proposed relay assisted cooperation scheme always increase the outage achievable rate (which is far from being the case for traditional orthogonal schemes). At low-to-moderate SNR, adding a relay bring substantial gains compare to no cooperation (MAC). However, we find a law of diminishing returns when we increase the number of relays. It militates for small cooperation structure with a few relays. At very high SNR, we notice two things: Firstly the NOMAMRC symmetric individual and common ϵ-outage achievable rate becomes equal to the MAC individual and common ϵ-outage one, respectively; Secondly, the MAC individual ϵ-outage achievable rate equals the common ϵ-outage achievable rate. The latter observation is demonstrated in appendix A, while the former is demonstrated in Appendix B (which builds on the previous one).

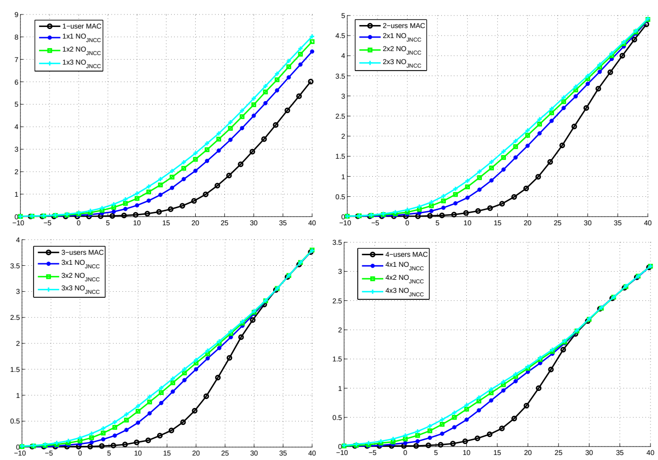

Fig. 2. The symmetric individual ϵ-outage achievable rate as a function of γ where $\epsilon = 0.01$, $M = 1, 2, 3, 4$, $L = 0, 1, 2, 3$, $\gamma_{rd} = \gamma$, $\gamma_{sr} = \gamma_{sd} = \alpha\gamma$, and $\alpha = 2/3$.

In the second set of simulation, we increase the reliability of the source-to-relay channels by adding 10db with respect to previous simulations. As a result, the average receive SNR over the different links become $\gamma_{rd} = \gamma$, $\gamma_{sr} = \alpha 10\gamma$, and $\gamma_{sd} = \alpha\gamma$. Fig. 4 shows the common ϵ-outage achievable rate for a number of relays ranging from $L = 0$ to $L = 4$ and $M = 4$. The more the source-to-relay channels are reliable, the more improvements in the ϵ-outage achievable rate we can obtain compared to no-cooperation. The behavior at very high SNR keeps the same, i.e., the ϵ-outage achievable rate of NOMAMRC converges towards the MAC ϵ-outage capacity (demonstrated in Appendix B).

Finally, in the third set of simulations, we vary the factor α to see its impact on the ϵ-outage achievable rate. Fig. 5 shows

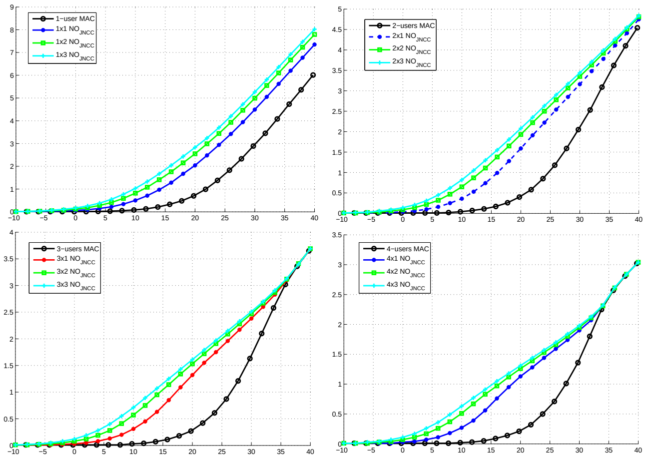

Fig. 3. The symmetric common ϵ-outage achievable rate as a function of γ where $\epsilon = 0.01$, $M = 1, 2, 3, 4$, $L = 0, 1, 2, 3$, $\gamma_{rd} = \gamma$, $\gamma_{sr} = \gamma_{sd} = \alpha\gamma$, and $\alpha = 2/3$.

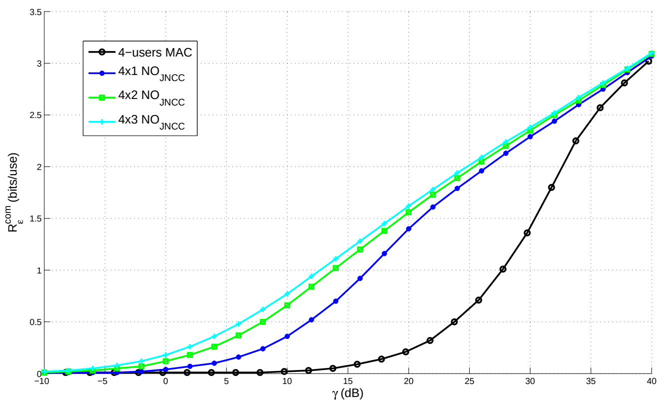

Fig. 4. The symmetric common ϵ-outage achievable rate as a function of γ where $\epsilon = 0.01$, $M = 4$, $L = 0, 1, 2, 3$, $\gamma_{rd} = \gamma$, $\gamma_{sr} = \alpha 10\gamma$, $\gamma_{sd} = \alpha\gamma$, and $\alpha = 2/3$.

the common ϵ-outage achievable rate for different values of α in the case of $M = 3$ and $L = 1$ given the same averaged received SNR for all links ($\gamma_{rd} = \gamma_{sr} = \gamma_{sd} = \gamma$). As previously stated in [7], $\alpha = 2/3$ is good choice for low to medium SNR regime. Once again, the high SNR regime follows the behavior explained in Appendix B.

V. CONCLUSIONS

In this paper, we have formulated the individual and common outage event for the relay assisted cooperative communication scheme NOMAMRC. We have illustrated that the NOMAMRC always outperforms the no-cooperation case even in presence of noisy slow fading source-to-relay links which is an extremely desirable feature. We have demonstrated that at high SNR regime the NOMAMRC converges to the same ϵ-outage achievable rate as the MAC. Future works will consider practical distributed coding and decoding designs (some have already been suggested in [7]) to reach as close as possible these outage rates under reasonable performance complexity trade offs.

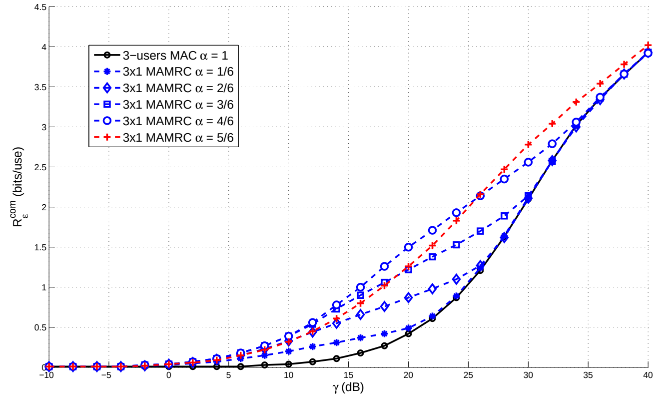

Fig. 5. The symmetric common ϵ-outage capacity for NOMAMRC/JNCC where $M = 3, L = 1, \epsilon = 0.01$, and $\gamma_{sr} = \gamma_{sd} = \gamma_{rd} = \gamma$, for different values of α.

APPENDIX A

In this Appendix, we demonstrate that the symmetric M-user MAC common and individual ϵ-outage achievable rate, i.e., given in (14) and (15), behave similarly at high SNR regime. The basic idea of the proof is suggested in [10, Exercice 6.14]. Let us consider a $|\mathcal{S}|$-user MAC with $\mathcal{S} = \{s_1, \ldots, s_{|\mathcal{S}|}\}$. We further assume that $\forall s \in \mathcal{S}$, $\gamma_{sd} = \gamma$ and define the normalized channel coefficients $c_{sd} = h_{sd}/\sqrt{\gamma}$. It yields from (7) that the common outage event is

$$\mathcal{E}_{d,\mathcal{S}} = \bigcup_{\mathcal{U}\subseteq\mathcal{S}} \{\sum_{s\in\mathcal{U}} |c_{sd}|^2 < \frac{2^{|\mathcal{U}|R} - 1}{\gamma}\} \tag{20}$$

From (11) the individual outage event of source s is given by

$$\mathcal{O}_{d,s} = \bigcap_{\mathcal{I}\subset\mathcal{S}: s\in\mathcal{I}^c} \bigcup_{\mathcal{U}\subseteq\mathcal{I}^c} \{\sum_{s\in\mathcal{U}} |c_{sd}|^2 < \frac{2^{|\mathcal{U}|R} - 1}{\gamma} \left(1 + \gamma \sum_{\tilde{s}\in\mathcal{I}} |c_{\tilde{s}d}|^2\right)\} \tag{21}$$

The sum rate constraint yields the sum rate upper bound conditional on an ϵ-outage probability

$$\frac{2^{|\mathcal{S}|R} - 1}{\gamma} \leq F^{-1}_{|\mathcal{S}|}(\epsilon). \tag{22}$$

where $F^{-1}_{|\mathcal{S}|}(x)$ is defined as the inverse function of the cumulative distribution function of the sum of $|\mathcal{S}|$ i.i.d exponential random variables with parameter 1, which is given by

$$F_{|\mathcal{S}|}(x) = 1 - e^{-x} \sum_{j=0}^{|\mathcal{S}|-1} \frac{x^j}{j!} \tag{23}$$

Let us check that indeed this upper bound is achieved and yields the common and individual ϵ-outage achievable rate when γ tends to infinity. If $\frac{2^{|\mathcal{S}|R}-1}{\gamma} = \delta$ where $\delta = F^{-1}_{|\mathcal{S}|}(\epsilon)$ for γ tending to infinity, it is easy to check that $\frac{2^{|\mathcal{U}|R}-1}{\gamma}$, $|\mathcal{U}| < |\mathcal{S}|$ goes to 0 and $\frac{2^{|\mathcal{U}|R}-1}{\gamma}\left(1 + \gamma \sum_{\tilde{s}\in\mathcal{I}} |c_{\tilde{s}d}|^2\right)$ goes to infinity almost surely. The former observation yields that the probability of the outage events $\mathcal{F}_{d,\mathcal{S}}(\mathcal{U}) = \{\sum_{s\in\mathcal{U}} |c_{sd}|^2 < \frac{2^{|\mathcal{U}|R}-1}{\gamma}\}$ in (20), is one when $|\mathcal{U}| < |\mathcal{S}|$. which is equivalent to state that $R_\epsilon^{com} = 1/|\mathcal{S}|\log(1+\gamma\delta)$. The latter observation means that an individual ϵ-outage achievable rate event always occurs when an common ϵ-outage achievable rate event occurs, i.e.,

$$\mathcal{O}_{d,s} = \mathcal{E}_{d,\mathcal{S}} = \mathcal{F}_{d,\mathcal{S}}(\mathcal{S}) \tag{24}$$

thus $R_\epsilon^{ind} = R_\epsilon^{com}$.

APPENDIX B

From Appendix A, the ϵ-outage common and individual achievable rate associated to the no-cooperation case is $R_\epsilon = 1/|\mathcal{S}|\log(1+\gamma\delta)$ where $\delta = F_{|\mathcal{S}|}^{-1}(\epsilon)$, at very high SNR. Let us prove that this achievable rate is indeed the achievable rate of the NOMAMRC. If $\frac{2^{|\mathcal{S}|R}}{\gamma}$ is fixed than $\frac{2^{|\mathcal{S}|R/\alpha}}{\gamma}$ tends to infinity when γ goes to infinity. Based on Appendix A, the individual outage probability of any source s at relay r is the same as the common outage probability at very high SNR. It comes that the individual outage probability $\Pr(\mathcal{O}_{r,s})$ that the source s cannot be decoded at the relay r is simply

$$\begin{aligned} \Pr(\mathcal{O}_{r,s}) &= \Pr(\mathcal{E}_{r,\mathcal{S}}) && (25)\\ &\stackrel{(a)}{\approx} \Pr(\sum_{s\in\mathcal{S}} |c_{sr}|^2 \leq \frac{2^{|\mathcal{S}|R/\alpha}}{\gamma}) && (26)\\ &= 1 && (27) \end{aligned}$$

where (a) is based on the fact that the $|\mathcal{S}|$-user MAC dominating event in the high SNR regime is the one associated to the sum rate constraint (see Appendix A). Since, $\forall s \in \mathcal{S}$ and $\forall r \in \mathcal{R}$, $\Pr(\mathcal{O}_{r,s}) = 1$, it yields that $\Pr(\mathcal{R}_a = \emptyset) = 1$ at very high SNR. Finally, we have demonstrated that for the high SNR regime, the ϵ-outage achievable rate of the NOMAMRC (R_ϵ^{ind} as well as R_ϵ^{com}) are the one of the $|\mathcal{S}|$-user MAC since the probability that each relay does not transmit is 1, i.e.,

$$R_\epsilon^{ind} = R_\epsilon^{com} = \frac{1}{|\mathcal{S}|}\log(1+\gamma F_{|\mathcal{S}|}^{-1}(\epsilon)) \tag{28}$$

REFERENCES

[1] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, 2nd ed. Wiley, 2006.

[2] R. Ahlswede, N. Cai, S.-Y. R. Li, , and R. W. Yeung, “Network information flow,” in *IEEE Trans. Inf. Theory*, vol. 46, Jul. 2000, p. 12041216.

[3] Z. Zhang and S. L. P. Lam, “Physical layer network coding,” in *Proc. ACM MOBICOM’06*, Sep. 2006.

[4] J. Laneman, D. Tse, and G. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behaviour,” *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.

[5] N. Prasad, G. Yue, X. Wang, , and M. Varanasi, “Optimal successive group decoders for mimo multiple-access channels,” *IEEE Trans. Inf. Theory*, vol. 54, 2008.

[6] S. Shamai and A. Steiner, “A broadcast approach for a single-user slowly fading mimo channel,” *IEEE Trans. Inf. Theory*, vol. 49, pp. 2617–2635, 2003.

[7] A. Hatefi, R. Visoz, and A. Berthet, “Near outage limit joint network coding and decoding for the non-orthogonal multiple-access relay channel,” in *Proc. IEEE PIMRC’12*, Sydney, Australia, Sep. 2012.

[8] E. Biglieri, J. Proakis, , and S. S. (Shitz), “Fading channels: Information-theoretic and communications aspects,” *IEEE Trans. Inf. Theory*, vol. 44, 1998.

[9] D. Duyck, D. Capirone, M. Moeneclaey, and J. Boutros, “A full-diversity joint network-channel code construction for cooperative communications,” in *Proc. IEEE PIMRC’09*, Tokyo, Japan, Sep. 2009.

[10] D. Tse, , and P. Viswanath, *Fundametals of Wireless Communication*. Cambridge University Press 2005.