MIMO Four-Way Relaying

(*Invited Paper*)

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Abstract*—Two-way relaying in wireless systems has initiated a large research effort during the past few years. Nevertheless, it represents only a specific traffic pattern and it is of interest to investigate other traffic patterns where such a simultaneous processing of information flows can bring performance advantage. In this paper we consider a *four-way relaying* multiple-input multiple-output (MIMO) scenario, where each of the two Mobile Stations (MSs) has a two-way connection to the same Base Station (BS), while each connection is through a dedicated Relay Station (RS). The RSs are placed in such a way that one RS and the terminals associated with it do not interfere with the other RS, and vice versa. We introduce and analyze a two-phase transmission scheme to serve the four-way traffic pattern defined in this scenario. Each phase consists of combined broadcast and multiple access. We analyze the sum-rate of the new scheme for Decode-and-Forward (DF) operational model for the RS. We compare the performance with state-of-the-art reference schemes, based on two-way relaying with DF. The results indicate that the sum-rate of the two-phase four-way relaying scheme largely outperforms the four-phase scheme, while closely approaching the performance achievable when the BS is replaced by two BSs.

I. Introduction

Wireless transceivers commonly work in a half-duplex mode, such that one-way relaying suffers from a loss in spectral efficiency as the relay cannot transmit and receive at the same time. This loss has been mitigated with the introduction of two-way relaying based on wireless network coding (WNC) [1]–[3]. Various aspects of two-way relaying have been investigated, such as achievable rates [4], optimal Decode-and-Forward (DF) broadcasting strategies [5] and differential modulation based Amplify-and-Forward (AF) strategies [6]. Paper [7] extends the optimal broadcasting into multiple-input multiple-output (MIMO) case.

In [8] we have leveraged the principles behind WNC to devise lattice-based Denoise-and-Forward transmission schemes for the four-way relay scenario, applicable to wireless cellular systems and depicted on Fig. 1. Each of the two Mobile Stations (MSs) U1 and U2 has a two-way connection to the same Base Station (BS), through a dedicated Relay Station (RS). The RSs are placed in such a way that one RS and the terminals associated with it do not interfere with the other RS, and vice versa. The state-of-the-art schemes for this scenario is time-division multiplexing two independent two-way relaying schemes in time as shown in Fig. 2(b). A similar scenario has recently been considered in [9], where DS-CDMA is used to avoid interference, the nodes use BPSK modulation and the relay applies physical-layer network coding.

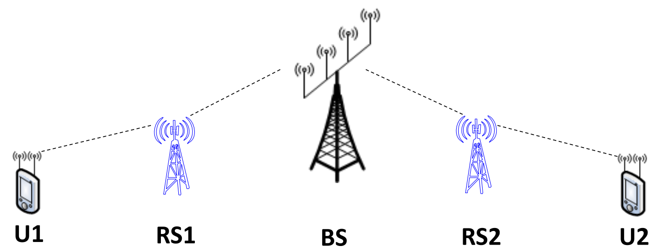
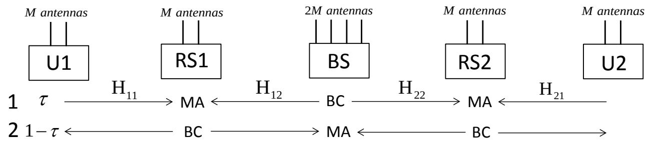


Figure 1. Four-way relaying with Base Station (BS), two Relay Stations RS1 and RS2, and two Mobile Stations (MSs) U1 and U2.

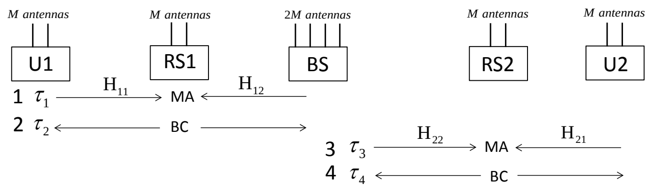
In [8] we have considered the four-way relaying scenario in which each node is equipped with a single antenna. In this paper we show that the use of multiple antennas at the BS and the RS is a highly non-trivial extension of the four-way relaying schemes described in [8]. Namely, a BS with a single antenna applies superposition coding in order to simultaneously send to both RSs. The use of multiple antennas at the BS opens the possibility to spatially separate the flows going to RS1 and RS2, respectively. The fact that the RSs are deployed in a way not to interfere with each other ensures that there is no loss of Degrees of Freedom (DoF) due to interference cancellation. We compare the sum-rate of the proposed two-phase four-way relaying scheme on Fig. 2(a) with two different reference schemes. The first is depicted on Fig. 2(b) and is the same physical setup as Fig. 2(a), as there is a single $2M$-antenna BS, but the communication on Fig. 2(a) is a time-division multiplexing of two schemes for two-way relaying. More interestingly, we compare the proposed scheme to the setup on Fig. 3, in which the two users are served by two different, non-interfering BSs, each with M antennas. The three schemes have identical DoF, but the numerical results show that the proposed scheme achieves similar sum-rate performance as the setup with two BSs on Fig. 3, while being superior to the scheme on Fig. 2(b). This implies that, practically, with the proposed scheme one BS can be saved.

II. System and Channel Models

We consider a two-way cellular network in which a BS exchanges information with the MSs U1 and U2 with the aid of RS1 and RS2, see Fig. 2(a). Each of the MS, RS has M antennas, while the BS has $2M$ antennas, therefore



(a) Four–way relaying transmission scheme consisting of two phases



(b) Four–way relaying transmission scheme consisting of four phases

Figure 2. Four–way relaying transmission schemes.

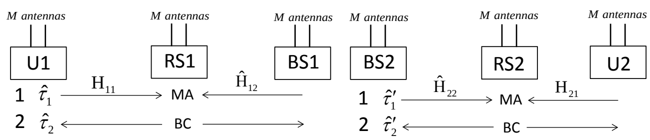


Figure 3. Two separate Two–way relaying transmission scheme.

ensuring that the DoF of the two flows are not decreased due to the BS. All nodes are half-duplex and no direct links are assumed between a MS and the BS. For simplicity, we use $1, R1, B, R2, 2$ as the indices to denote U1, RS1, BS, RS2 and U2, respectively. The channels are denoted by $\mathbf{H}_{11} \in \mathbb{C}^{[M\times M]}$(U1-RS1), $\mathbf{H}_{12} \in \mathbb{C}^{[M\times 2M]}$(BS-RS1), $\mathbf{H}_{22} \in \mathbb{C}^{[M\times 2M]}$(BS-RS2) and $\mathbf{H}_{21} \in \mathbb{C}^{[M\times M]}$(U2-RS2).

Fig. 2(b) illustrates a four-way relaying scheme based on time-division between two different two-way relaying instances. The first phase is the multiple access (MA) from U1 and BS to RS1. The second phase is the broadcast (BC) from RS1 to U1 and BS. The third and fourth phases are similar. Fig. 3 shows another benchmark, where there are two simultaneous but non-interfering two-way relaying transmissions with two BSs. For a fair comparison, each of the BSs on Fig. 3 uses M antennas. Similar to Fig. 2, the channels are denoted by $\mathbf{H}_{11} \in \mathbb{C}^{[M\times M]}$(U1-RS1), $\hat{\mathbf{H}}_{12} \in \mathbb{C}^{[M\times M]}$(BS-RS1), $\hat{\mathbf{H}}_{22} \in \mathbb{C}^{[M\times M]}$(BS-RS2) and $\mathbf{H}_{21} \in \mathbb{C}^{[M\times M]}$(U2-RS2). In all cases the relay uses DF operation.

Each channel in Fig. 2 and Fig. 3, is known at all the nodes. Each MS has a two-way, uplink/downlink traffic to/from the BS. $\mathbf{z}_k \sim \mathcal{CN}(0,1), k \in \{1, R1, B, R2, 2\}$ represents a vector containing circularly symmetric white Gaussian noise with covariance $\mathbf{R}_{\mathbf{z}_k\mathbf{z}_k^H} = \mathbf{I}$, where $\{\cdot\}^H$ stands for conjugate transpose. We assume the source information is normalized $\mathbb{E}\left\{\mathbf{x}_k\mathbf{x}_k^H\right\} = \mathbf{I}$. The maximal transmission power of node k is P_k which is decided by beamforming matrix or power allocation matrix.

III. Capacity region constraints

A. Capacity region constraints of two-phase four-way relaying

The downlink data at BS is $\mathbf{x}_B = [\mathbf{x}_{B1}, \mathbf{x}_{B2}]$ while $\mathbf{x}_{Bi} \in \mathbb{C}^{[M\times 1]}$ is intended to Ui, $i = 1, 2$. Zero-forcing (ZF) beamforming is applied at BS to avoid the mutual interference between $\mathbf{x}_{B1}$ and $\mathbf{x}_{B2}$ at RSs. So we have $\mathbf{H}_{22}\mathbf{W}_{B1} = \mathbf{H}_{12}\mathbf{W}_{B2} = 0$, here $\mathbf{W}_{Bi} \in \mathbb{C}^{[2M\times M]}, i = 1, 2$ is the beamforming matrix of $\mathbf{x}_{B1}$ and $\mathbf{x}_{B2}$. Therefore, the columns inside $\mathbf{W}_{Bi}$ are orthogonal to each other due to the ZF operation. We assume $\mathbf{P}_{Bi} = \mathrm{diag}([\mathbf{p}_{Bi}])$ as the two power allocation matrices at BS, where $\mathbf{p}_{Bi} \in \mathbb{C}^{[M\times 1]}$. To prevent $\mathbf{W}_{Bi}$ from affecting the power allocation, $\mathbf{W}_{Bi}\mathbf{W}_{Bi}^H = \mathbf{I}$ needs to be satisfied. Then the transmitting signal from the BS is $\mathbf{W}_{B1}(\mathbf{P}_{B1})^{\frac{1}{2}}\mathbf{x}_{B1} + \mathbf{W}_{B2}(\mathbf{P}_{B2})^{\frac{1}{2}}\mathbf{x}_{B2}$. The total transmission power at the BS has to satisfy $Tr\{\mathbf{P}_{B1}\} + Tr\{\mathbf{P}_{B2}\} \le P_B$.

The uplink data from Ui is $\mathbf{x}_i \in \mathbb{C}^{[M\times 1]}$. We assume $\mathbf{W}_i \in \mathbb{C}^{[M\times M]}$ as the beamforming matrix of Ui. Note that $\mathbf{W}_i$ is similar to $\mathbf{W}_{Bi}(\mathbf{P}_{Bi})^{\frac{1}{2}}$ at BS, such that $\mathbf{W}_i$ takes charge of the beamforming as well as the power allocation at the MS. Then the transmitting signal from Ui is $\mathbf{W}_i\mathbf{x}_i$. The covariance matrix is then $\mathbb{E}\left\{\mathbf{W}_i\mathbf{x}_i(\mathbf{W}_i\mathbf{x}_i)^H\right\} = \mathbf{Q}_i$ with transmission power constraint $Tr\{\mathbf{Q}_i\} \le P_i$. At the end of phase 1, due to the ZF principle at the BS, RSi receives

$$\mathbf{y}_{Ri} = \mathbf{H}_{i2}\mathbf{W}_{Bi}(\mathbf{P}_{Bi})^{\frac{1}{2}}\mathbf{x}_{Bi} + \mathbf{H}_{i1}\mathbf{W}_i\mathbf{x}_i + \mathbf{z}_{Ri}, i = 1, 2. \quad (1)$$

Define R_i^d as the downlink data rate of Ui and define R_i^u as the uplink data rate of Ui. From [10], the capacity region of the MIMO MA channel in (1) is

$$\begin{cases} R_i^u \le \tau\log_2\left|\mathbf{I}_M + \mathbf{H}_{i1}\mathbf{Q}_i\mathbf{H}_{i1}^H\right| & \text{(2a)} \\ R_i^d \le \tau\log_2\left|\mathbf{I}_M + \mathbf{H}_{i2}\mathbf{W}_{Bi}\mathbf{P}_{Bi}\mathbf{W}_{Bi}^H\mathbf{H}_{i2}^H\right| & \text{(2b)} \\ R_i^u + R_i^d \le \tau\log_2\left|\mathbf{I}_M + \mathbf{H}_{i1}\mathbf{Q}_i\mathbf{H}_{i1}^H\right. & \text{(2c)} \\ \qquad\left. +\mathbf{H}_{i2}\mathbf{W}_{Bi}\mathbf{P}_{Bi}\mathbf{W}_{Bi}^H\mathbf{H}_{i2}^H\right|,\ i = 1, 2 & \end{cases}$$

where τ is the duration of phase 1, and the hence the duration of phase 2 is $1-\tau$. The whole capacity region is the union of the capacity region at all power allocation setting points. In the following, for simplicity, we also only present the capacity region, when the power allocation and/or the input covariance matrices are fixed. The optimization regarding the related variables is covered in Section IV.

RSi decodes the signal $\mathbf{x}_i$ and $\mathbf{x}_{Bi}$, then re-encodes the messages of $\mathbf{x}_i$ and $\mathbf{x}_{Bi}$ into $\mathbf{x}_{Ri} \in \mathbb{C}^{[M\times 1]}$. We assume $\mathbf{T}_{Ri} \in \mathbb{C}^{[M\times M]}$ to be the beamforming matrix at RSi with covariance matrix $\mathbf{Q}_{Ri} = \mathbb{E}\left\{\mathbf{T}_{Ri}\mathbf{x}_{Ri}(\mathbf{T}_{Ri}\mathbf{x}_{Ri})^H\right\}$ and the transmission power constraint is imposed as $Tr\{\mathbf{Q}_{Ri}\} \le P_{Ri}$. At the end of phase 2, the received signals at the two MSs and the BS are

$$\mathbf{y}_i = \mathbf{H}_{i1}^{\mathrm{H}}\mathbf{T}_{Ri}\mathbf{x}_{Ri} + \mathbf{z}_i,\ \mathbf{y}_B = \sum_{i=1}^{2}\mathbf{H}_{i2}^H\mathbf{T}_{Ri}\mathbf{x}_{Ri} + \mathbf{z}_B \quad (3)$$

where $\mathbf{y}_i$ is received by Ui and $\mathbf{y}_B$ is received by the BS. Equation (3) describes the combination of a BC and a MA channel, both with side information: the BC channel from RSi to Ui and BS and the MA channel at BS with signals from RS1 and RS2. When the input covariance matrices are fixed, the capacity region of this combined MIMO BC and MA channels in (3) is (see Appendix A for the derivation):

$$\begin{cases} R_i^d \leq (1-\tau)\log_2\left|\mathbf{I}_M + \mathbf{H}_{i1}^{\mathrm{H}}\mathbf{Q}_{Ri}\mathbf{H}_{i1}\right| & \text{(4a)} \\ R_i^u \leq (1-\tau)\log_2\left|\mathbf{I}_{2M} + \mathbf{H}_{i2}^{H}\mathbf{Q}_{Ri}\mathbf{H}_{i2}\right| & \text{(4b)} \\ R_1^u + R_2^u \leq (1-\tau)\log_2\left|\mathbf{I}_{2M} + \sum\limits_{i=1,2}\mathbf{H}_{i2}^{H}\mathbf{Q}_{Ri}\mathbf{H}_{i2}\right| & \text{(4c)} \end{cases}$$

B. Rate region constraints of four-phase four-way relaying

We use $\tau_1, \tau_2, \tau_3, \tau_4$ to denote the time ratios spent by phase 1,2,3,4, respectively. Notice that, $\tau_1+\tau_2+\tau_3+\tau_4 = 1$ must be satisfied. We assume $\tilde{\mathbf{W}}_{Bi} \in \mathbb{C}^{[2M\times 2M]}$ as the beamforming matrix for $\tilde{\mathbf{x}}_{Bi} \in \mathbb{C}^{[2M\times 1]}$ at BS with covariance matrix $\mathbb{E}\left\{\tilde{\mathbf{W}}_{Bi}\tilde{\mathbf{x}}_{Bi}\left(\tilde{\mathbf{W}}_{Bi}\tilde{\mathbf{x}}_{Bi}\right)^H\right\} = \tilde{\mathbf{Q}}_{Bi}$ with the transmission power $Tr\left\{\tilde{\mathbf{Q}}_{Bi}\right\} \leq P_B$. The definition of beamforming matrices and covariance matrices at RSs and MSs remains the same as in section III-A. At the end of phase 1, RS1 receives

$$\mathbf{y}_{R1} = \mathbf{H}_{12}\tilde{\mathbf{W}}_{B1}\tilde{\mathbf{x}}_{B1} + \mathbf{H}_{11}\mathbf{W}_1\mathbf{x}_1 + \mathbf{z}_{R1}, i = 1,2. \quad (5)$$

From [10], when the power allocation and the input covariance matrices are fixed, the rate region of the MIMO MA channel in (5) is

$$\begin{cases} R_1^u \leq \tau_1\log_2\left|\mathbf{I}_M + \mathbf{H}_{11}\mathbf{Q}_1\mathbf{H}_{11}^H\right| & \text{(6a)} \\ R_1^d \leq \tau_1\log_2\left|\mathbf{I}_M + \mathbf{H}_{12}\tilde{\mathbf{Q}}_{B1}\mathbf{H}_{12}^H\right| & \text{(6b)} \\ R_1^u + R_1^d \leq \tau_1\log_2\left|\mathbf{I}_M + \mathbf{H}_{11}\mathbf{Q}_1\mathbf{H}_{11}^H + \mathbf{H}_{12}\tilde{\mathbf{Q}}_{B1}\mathbf{H}_{12}^H\right|. & \text{(6c)} \end{cases}$$

At the end of phase 2, the received signals are

$$\mathbf{y}_1 = \mathbf{H}_{11}^H\mathbf{T}_{R1}\mathbf{x}_{R1} + \mathbf{z}_1,\ \mathbf{y}_B = \mathbf{H}_{12}^H\mathbf{T}_{R1}\mathbf{x}_{R1} + \mathbf{z}_B. \quad (7)$$

where $\mathbf{y}_1$ is received by U1 and $\mathbf{y}_B$ is received by the BS. From [7], when the input covariance matrices are fixed, the rate region of MIMO BC channel in (7) is

$$\begin{cases} R_1^d \leq \tau_2\log_2\left|\mathbf{I}_M + \mathbf{H}_{11}^H\mathbf{Q}_{R1}\mathbf{H}_{11}\right| & \text{(8a)} \\ R_1^u \leq \tau_2\log_2\left|\mathbf{I}_{2M} + \mathbf{H}_{12}^H\mathbf{Q}_{R1}\mathbf{H}_{12}\right|. & \text{(8b)} \end{cases}$$

Similar to phase 1 and phase 2, the rate region of phase 3 and phase 4 is

$$\begin{cases} R_2^u \leq \tau_3\log_2\left|\mathbf{I}_M + \mathbf{H}_{21}\mathbf{Q}_2\mathbf{H}_{21}^H\right| & \text{(9a)} \\ R_2^d \leq \tau_3\log_2\left|\mathbf{I}_M + \mathbf{H}_{22}\tilde{\mathbf{Q}}_{B2}\mathbf{H}_{22}^H\right| & \text{(9b)} \\ R_2^u + R_2^d \leq \tau_3\log_2\left|\mathbf{I}_M + \mathbf{H}_{21}\mathbf{Q}_2\mathbf{H}_{21}^H + \mathbf{H}_{22}\tilde{\mathbf{Q}}_{B2}\mathbf{H}_{22}^H\right| & \text{(9c)} \\ R_2^d \leq \tau_4\log_2\left|\mathbf{I}_M + \mathbf{H}_{21}^H\mathbf{Q}_{R2}\mathbf{H}_{21}\right| & \text{(9d)} \\ R_2^u \leq \tau_4\log_2\left|\mathbf{I}_{2M} + \mathbf{H}_{22}^H\mathbf{Q}_{R2}\mathbf{H}_{22}\right|. & \text{(9e)} \end{cases}$$

C. Rate region constraints of two separate two-way relaying

We use $\hat{\tau}_1$ and $\hat{\tau}_2$ to denote the time ratios spent by phase 1 and phase 2 of the first two-way relaying and use $\hat{\tau}_1'$ and $\hat{\tau}_2'$ to denote the time ratios spent by phase 1 and phase 2 of the second two-way relaying. We assume $\hat{\mathbf{W}}_{Bi} \in \mathbb{C}^{[M\times M]}$ as the beamforming matrix of $\hat{\mathbf{x}}_{Bi} \in \mathbb{C}^{[M\times 1]}$ at BSi with covariance matrix $\mathbb{E}\left\{\hat{\mathbf{W}}_{Bi}\hat{\mathbf{x}}_{Bi}\left(\hat{\mathbf{W}}_{Bi}\hat{\mathbf{x}}_{Bi}\right)^H\right\} = \hat{\mathbf{Q}}_{Bi}$ with the transmission power budget $Tr\left\{\hat{\mathbf{Q}}_{Bi}\right\} \leq P_B$. The definition of beamforming matrix and covariance matrix at RSs and MSs is the same as section III-A. When the power allocation and the input covariance matrices are fixed, the capacity region can be obtained similar to III-B as follows:

$$\begin{cases} R_1^u \leq \hat{\tau}_1\log_2\left|\mathbf{I}_M + \mathbf{H}_{11}\mathbf{Q}_1\mathbf{H}_{11}^H\right| & \text{(10a)} \\ R_1^d \leq \hat{\tau}_1\log_2\left|\mathbf{I}_M + \hat{\mathbf{H}}_{12}\hat{\mathbf{Q}}_{B1}\hat{\mathbf{H}}_{12}^H\right| & \text{(10b)} \\ R_1^u + R_1^d \leq \hat{\tau}_1\log_2\left|\mathbf{I}_M + \mathbf{H}_{11}\mathbf{Q}_1\mathbf{H}_{11}^H + \hat{\mathbf{H}}_{12}\hat{\mathbf{Q}}_{B1}\hat{\mathbf{H}}_{12}^H\right| & \text{(10c)} \\ R_1^d \leq \hat{\tau}_2\log_2\left|\mathbf{I}_M + \mathbf{H}_{11}^H\mathbf{Q}_{R1}\mathbf{H}_{11}\right| & \text{(10d)} \\ R_1^u \leq \hat{\tau}_2\log_2\left|\mathbf{I}_M + \hat{\mathbf{H}}_{12}^H\mathbf{Q}_{R1}\hat{\mathbf{H}}_{12}\right|. & \text{(10e)} \end{cases}$$

$$\begin{cases} R_2^u \leq \hat{\tau}_1'\log_2\left|\mathbf{I}_M + \mathbf{H}_{21}\mathbf{Q}_2\mathbf{H}_{21}^H\right| & \text{(11a)} \\ R_2^d \leq \hat{\tau}_1'\log_2\left|\mathbf{I}_M + \hat{\mathbf{H}}_{22}\hat{\mathbf{Q}}_{B2}\hat{\mathbf{H}}_{22}^H\right| & \text{(11b)} \\ R_2^u + R_2^d \leq \hat{\tau}_1'\log_2\left|\mathbf{I}_M + \mathbf{H}_{21}\mathbf{Q}_2\mathbf{H}_{21}^H + \hat{\mathbf{H}}_{22}\hat{\mathbf{Q}}_{B2}\hat{\mathbf{H}}_{22}^H\right| & \text{(11c)} \\ R_2^d \leq \hat{\tau}_2'\log_2\left|\mathbf{I}_M + \mathbf{H}_{21}^H\mathbf{Q}_{R2}\mathbf{H}_{21}\right| & \text{(11d)} \\ R_2^u \leq \hat{\tau}_2'\log_2\left|\mathbf{I}_M + \hat{\mathbf{H}}_{22}^H\mathbf{Q}_{R2}\hat{\mathbf{H}}_{22}\right|. & \text{(11e)} \end{cases}$$

where $\hat{\tau}_1 + \hat{\tau}_2 = 1$ and $\hat{\tau}_1' + \hat{\tau}_2' = 1$ indicate that the two separate two-way relay transmissions occur simultaneously.

IV. Weighted sum-rate optimization

We focus on the weighted sum-rate maximization. If the rate region is convex, a rate tuple on the boundary of the rate region is a solution of a weighted rate-sum problem. We assume $w_1^u, w_1^d, w_2^u, w_2^d$ as the weights for $R_1^u, R_1^d, R_2^u, R_2^d$ respectively, which act as the rate awards for individual information flows. If $w_1^u = w_1^d = w_2^u = w_2^d$, the weighted sum-rate problem degrades to the sum-rate problem. All the solutions from $w_1^u + w_1^d + w_2^u + w_2^d = 1$ compose the boundary of the rate region of the whole system [11]. With the rate region constraints in III-A, the weighted sum-rate optimization problem of two-phase four-way relaying can be formulated as

$$\begin{aligned} \max_{\tau,\mathbf{P}_{Bi},\mathbf{Q}_i,\mathbf{Q}_{Ri}} \quad & w_1^u R_1^u + w_1^d R_1^d + w_2^u R_2^u + w_2^d R_2^d \\ s.t. \quad & (2),\ (4),\ \mathbf{Q}_i, \mathbf{Q}_{Ri} \succeq 0,\ \mathbf{P}_{Bi} \geq 0 \\ & Tr\left\{\mathbf{Q}_i\right\} \leq P_i, \quad Tr\left\{\mathbf{Q}_{Ri}\right\} \leq P_{Ri} \\ & Tr\left\{\mathbf{P}_{B1}\right\} + Tr\left\{\mathbf{P}_{B2}\right\} \leq P_B \end{aligned} \quad (12)$$

where $\succeq$ denotes the semidefinite requirement. Note that $\mathbf{W}_{Bi}$ is given from the ZF operation at the BS. The optimization focuses on the input covariance matrices at the MSs and the relays, the power allocation matrices at the BS, and the time division between the two phases. Similarly, we can formulate the weighted sum-rate optimization of four-phase four-way relaying and two separate two-way relaying as (13) and (14) respectively.

$$\begin{aligned}&\max_{\tau_1,\tau_2,\tau_3,\tau_4,\mathbf{Q}_i,\tilde{\mathbf{Q}}_{Bi},\mathbf{Q}_{Ri}} \quad w_1^u R_1^u + w_1^d R_1^d + w_2^u R_2^u + w_2^d R_2^d \\ &s.t.\ \ (6),\ (8),\ (9),\ \tau_1+\tau_2+\tau_3+\tau_4=1 \\ &\mathbf{Q}_i, \tilde{\mathbf{Q}}_{Bi}, \mathbf{Q}_{Ri} \succeq 0,\quad Tr\left\{\mathbf{Q}_i\right\} \le P_i \\ &Tr\left\{\tilde{\mathbf{Q}}_{Bi}\right\} \le P_B,\quad Tr\left\{\mathbf{Q}_{Ri}\right\} \le P_{Ri}\end{aligned} \tag{13}$$

$$\begin{aligned}&\max_{\hat{\tau}_1,\hat{\tau}_2,\hat{\tau}_1',\hat{\tau}_2',\mathbf{Q}_i,\hat{\mathbf{Q}}_{Bi},\mathbf{Q}_{Ri}} \quad w_1^u R_1^u + w_1^d R_1^d + w_2^u R_2^u + w_2^d R_2^d \\ &s.t.\ \ (10),\ (11),\ \hat{\tau}_1+\hat{\tau}_2=1,\ \hat{\tau}_1'+\hat{\tau}_2'=1 \\ &\mathbf{Q}_i, \hat{\mathbf{Q}}_{Bi}, \mathbf{Q}_{Ri} \succeq 0,\quad Tr\left\{\mathbf{Q}_i\right\} \le P_i \\ &Tr\left\{\hat{\mathbf{Q}}_{Bi}\right\} \le P_B,\quad Tr\left\{\mathbf{Q}_{Ri}\right\} \le P_{Ri}\end{aligned} \tag{14}$$

The three problems (12), (13) and (14) are non-convex in general [12]. We resort to the alternating optimization framework here in order to obtain at least a local optimum solution for the weighted sum-rate maximization. Let us take the problem in (12) as an example to illustrate the alternating principle:

- If we fix $\mathbf{P}_{Bi}, \mathbf{Q}_i, \mathbf{Q}_{Ri}$, the only variable is τ. The optimization degrades to the linear programming (LP) optimization, which is also a convex problem [12].
- If we fix τ, the rest variables are $\mathbf{P}_{Bi}, \mathbf{Q}_i, \mathbf{Q}_{Ri}$. It is easy to prove that this is a convex optimization problem [12].

Based on the above observations, we apply the iterative algorithm shown in Algorithm 1 to solve the problem in (12), via fixing τ to solve $\mathbf{P}_{Bi}, \mathbf{Q}_i, \mathbf{Q}_{Ri}$ then fixing $\mathbf{P}_{Bi}, \mathbf{Q}_i, \mathbf{Q}_{Ri}$ to solve τ through the alternating steps. Note the initial value for problem (12) is chosen to be $\tau = 0.5$.

Algorithm 1 Alternating Optimization

set $n = 0$ and $\tau^{(0)} = 0.5$
iterate
  update $n = n + 1$
  1) compute $\mathbf{P}_{Bi}^{(n)}, \mathbf{Q}_i^{(n)}, \mathbf{Q}_{Ri}^{(n)}$ given $\tau^{(n-1)}$ using standard convex problem solver
  2) compute $\tau^{(n)}$ given $\mathbf{P}_{Bi}^{(n)}, \mathbf{Q}_i^{(n)}, \mathbf{Q}_{Ri}^{(n)}$ using standard convex problem solver
until weighted sum-rate convergence

Similar iterative algorithms also can be applied into (13) and (14), hence the details are neglected here due to space limitation. The iterative algorithms guarantee to converge with small number of iterations [13].

V. Numerical results

In the simulation, we assume each channel element obeys the complex Gaussian distribution: the real and imaginary parts of the antenna-to-antenna channel are zero mean and unit variance Gaussian random variables. For simplicity, we set $M = 2$ and each node has the same maximal power P which is the horizontal axis in Fig. 4. To ensure the convergence, the sum-rate is the average of 4000 channel realization results.

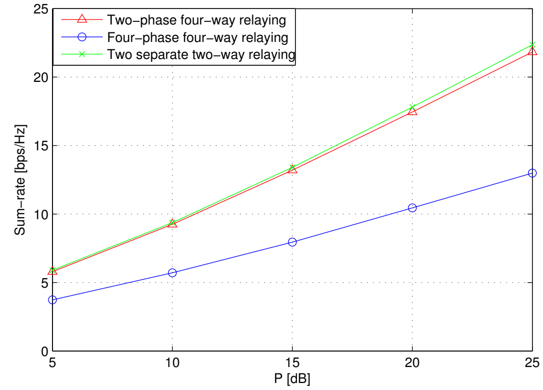


Figure 4. Sum-rate comparison of the proposed scheme with the two reference schemes.

Notice that, all the schemes on Fig. 4 support 8 streams in all, so these 3 schemes have the same DoF. But the sum-rates are quite different for the three schemes. The sum-rate of four-phase four-way relaying is the lowest as it does not fully exploit the spatial resources. Two separate two-way relaying achieves the highest sum-rate by the cost of 2 BSs. It should be noted that in the proposed two-phase four-way relaying, a simple ZF beamforming is applied at the BS to avoid the potential interference at the RSs. Yet, the efficient suboptimal BC beamforming with a single BS results in a near-optimal weighted sum-rate performance, very close to two spatially separated two-way relaying processes that use two BSs.

VI. Conclusion

We have described a new multi-way relay MIMO scenario of practical relevance, termed four-way relaying, in which each of the two MSs has a two-way connection to the same BS, while each connection is through a dedicated RS. The main assumption is that the two RSs are deployed with ideal spatial reuse and do not interfere with each other. We have proposed a novel communication scheme for serving two two-way traffic flows with multi-antenna BS, RS, and MS. We have compared the scheme to two reference schemes, each of them based on two-way relaying. The proposed scheme is superior in terms of achievable sum-rate compared to the reference scheme that is based on the same physical setup, but serves the flows by time-mulitplexing of two two-way relaying scheme. The most interesting result is that the sum-rate of the proposed scheme closely approaches the sum-rate of the scheme that uses two BSs with ideal spatial reuse, implying that the proposed scheme practically saves one BS. In our future work we will analyze the proposed scheme with practical path loss coefficients and multiple cells, in order to evaluate the overall improvement in spatial reuse.

Appendix A

For each RS in Fig. 2(a), the transmission in phase 2 is a BC process with side information at the receivers. On the other hand, the simultaneous transmission of RS1 and RS2 defines a MA channel at the BS. In principle, this makes the achievable rates of the two BC transmissions interrelated. We prove that

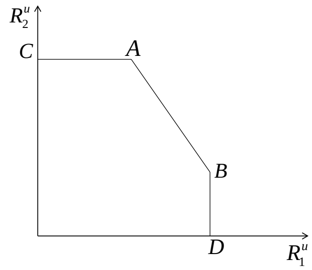


Figure 5. The capacity region of MIMO MAC with fixed covariance matrices

the achievable rate region can actually be decomposed in 2 independent region: 1) a MA region through channels $\mathbf{H}_{12}^H$ and $\mathbf{H}_{22}^H$ for rates (R_1^u, R_2^u), independent from the links $\mathbf{H}_{11}^H$ and $\mathbf{H}_{21}^H$ and 2) 2 single link rate regions for rates R_1^d and R_2^d through $\mathbf{H}_{11}^H$ and $\mathbf{H}_{21}^H$.

From [10], the MIMO MA region is equal to the union of pentagons and each pentagon corresponds to a different set of transmit covariance matrices. For fixed power allocation, consider the MA rate region through channels $\mathbf{H}_{12}^H$ and $\mathbf{H}_{22}^H$ in Fig. 5. We define $A = (R_1^u(A), R_2^u(A))$ and $B = (R_1^u(B), R_2^u(B))$ as the corner points of the MA region, where $R_1^u(A) = \log_2 \left|\mathbf{I}_{2\mathrm{M}} + \mathbf{H}_{12}^H \mathbf{Q}_{R1} \mathbf{H}_{12} \left(\mathbf{H}_{22}^H \mathbf{Q}_{R2} \mathbf{H}_{22} + \mathbf{I}_{2\mathrm{M}}\right)^{-1}\right|$, $R_2^u(A) = \log_2 \left|\mathbf{I}_{2\mathrm{M}} + \mathbf{H}_{22}^H \mathbf{Q}_{R2} \mathbf{H}_{22}\right|$, $R_1^u(B) = \log_2 \left|\mathbf{I}_{2\mathrm{M}} + \mathbf{H}_{12}^H \mathbf{Q}_{R1} \mathbf{H}_{12}\right|$, $R_2^u(B) = \log_2 \left|\mathbf{I}_{2\mathrm{M}} + \mathbf{H}_{22}^H \mathbf{Q}_{R2} \mathbf{H}_{22} \left(\mathbf{H}_{12}^H \mathbf{Q}_{R1} \mathbf{H}_{12} + \mathbf{I}_{2\mathrm{M}}\right)^{-1}\right|$. In the following we prove that point A can be achieved as well as the capacity of individual link $\mathbf{H}_{i1}^H$. Likewise, point B can be achieved. Time-sharing achieves the region border between A and B.

We recall that $\mathbf{x}_{Ri}$ is the re-encoded codeword at RSi. The BS receives both codewords $\mathbf{x}_{R1}$ and $\mathbf{x}_{R2}$ though channels $\mathbf{H}_{12}^H$ and $\mathbf{H}_{22}^H$. Let us consider a strategy where $\mathbf{x}_{R2}$ is treated as noise in order to decode $\mathbf{x}_{R1}$. Then, transmission from RS1 is a BC process with side information at the receivers U1 and BS. From [7], for fixed power allocation setting, the achievable upper bound on R_1^d on link $\mathbf{H}_{11}^H$ is $\log_2 \left|\mathbf{I}_M + \mathbf{H}_{11}^{\mathrm{H}} \mathbf{Q}_{R1} \mathbf{H}_{11}\right|$ (denoted as C_{11}) while the achievable upper bound on R_1^u on link $\mathbf{H}_{12}^H$ is $R_1^u(A)$. After decoding $\mathbf{x}_{R1}$, BS cancels its contribution from the received signal. Then transmission from RS2 is a BC process with side information at the receivers U2 and BS. From [7], for fixed power allocation setting, the achievable upper bound on R_2^u on link $\mathbf{H}_{22}^H$ is $R_2^u(A)$ while the achievable upper bound on R_2^d on link $\mathbf{H}_{21}^H$ is the capacity $\log_2 \left|\mathbf{I}_M + \mathbf{H}_{21}^{\mathrm{H}} \mathbf{Q}_{R2} \mathbf{H}_{21}\right|$ (denoted as C_{21}).

We have so far proved that the MA region corner point A can be achieved as well as the capacity of the link $\mathbf{H}_{i1}^H$. We denote $\mathbf{x}_{Ri}^A$ as corresponding optimal codewords. In a similar manner we can prove that the rates corresponding to the point B can be achieved. We denote $\mathbf{x}_{Ri}^B$ as optimal codewords. To summarize, an achievable upper bound on the rate tuple $(R_1^u, R_2^u, R_1^d, R_2^d)$ is $(R_1^u(A), R_2^u(A), C_{11}, C_{21})$ and is achieved by codewords $\mathbf{x}_{Ri}^A$. Another achievable upper bound is $(R_1^u(B), R_2^u(B), C_{11}, C_{21})$ is achieved by codewords $\mathbf{x}_{Ri}^B$.

The rates on the line AB in Fig. 5 are achieved by time-sharing between the codewords $\mathbf{x}_{Ri}^A$ and $\mathbf{x}_{Ri}^B$. Since U1, U2 and BS know the exact duration of $\mathbf{x}_{Ri}^A$ and $\mathbf{x}_{Ri}^B$ in the time-sharing codeword, the decoding process during t is to decode $\mathbf{x}_{Ri}^A$, the decoding process during $1-t$ is to decode $\mathbf{x}_{Ri}^B$. Therefore, $(tR_1^u(A) + \bar{t}R_1^u(B), tR_2^u(A) + \bar{t}R_2^u(B), C_{11}, C_{21})$ is achievable for every $t \in [0,1]$, $\bar{t} = 1-t$. This proves that the line AB in Fig. 5, is achievable while the single-user capacity of the channel with $\mathbf{H}_{i1}^H$ can also be achieved. Note that the time-sharing does not affect the decoding at Ui, since Ui knows the detailed structure of the re-encoded codeword, such that it can apply the appropriate side information. Similarly, by time-sharing between corner points and axes points, the rates on the lines CA and BD in Fig. 5 are also achievable. Then the whole MA region in Fig. 5 is achievable while the capacity of link $\mathbf{H}_{i1}^H$ is achievable. We thus obtain (4) where $1-\tau$ corresponds to the duration of phase 2.

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