Abstract—Relay deployments promise to alleviate the limitations of conventional macrocell networks, such as poor indoor penetration and coverage holes in a cost-efficient way. In this context, the capacity of the wireless relay link between a relay node (RN) and its serving base station (BS) has a crucial role in the achievable end-to-end performance. The deployment flexibility of RNs, which mainly stems from the wireless relay link, compact physical characteristics, and low-power consumption, can be exploited by relay site planning (RSP) to overcome the limitations of the relay link and, thus, enhance the system performance. To this end, RSP is carried out via selecting an RN deployment location from a discrete set of alternatives considering the signal-to-interference-plus-noise ratio (SINR) on the relay link as the selection criterion. In practice, the so-called coarse RSP takes into account only large-scale fading due to shadowing. Nevertheless, as RNs are stationary, the wireless channels pertaining to relay deployments are subject to simultaneous impairments by both shadowing and multi-path fading, i.e., composite fading/shadowing. In this paper, we present the performance of coarse RSP that can be used for planning and dimensioning of two-hop cellular relay networks in composite fading/shadowing environments, where co-channel interference is also present. The relay link is modeled by Nakagami-lognormal fading/shadowing channel interference. That is, therein, the maximum achievable gains through site location selection are presented, where the optimal RSP takes into account both shadowing and multi-path fading. Nevertheless, in practice, due to changes in multi-path fading because of, e.g., moving scatterers on the ground, the resultant RSP gains can decrease.

In this paper, we place the focus on the coarse RSP, which considers only shadowing for site location selection. The corresponding performance is evaluated in composite fading/shadowing environments with co-channel interference. We emphasize that composite fading/shadowing is frequently experienced especially in scenarios with low or no mobility [10], [11]. In addition, given the full-frequency reuse in future cellular networks, co-channel interference is another vital factor to be taken into account for accurate performance analysis. On this basis, we demonstrate the achievable SINR gains on the relay link by the coarse RSP. Though these gains deviate from the maximum achievable levels, results still show a clear gain on the relay link especially when multi-path fading is not severe. Also, the gains translate into higher achieved end-to-end rate provided that the system is not limited by the access link. Consequently, the performance improvements...
justify the use of coarse RSP in cellular relay networks.

The remainder of the paper is organized as follows. Section II briefly presents the channel models. In Section III, the modeling of RSP is summarized. In Section IV, the impact of RSP is analyzed along with SINR and rate derivations. Performance results and evaluations are provided in Section V. Finally, Section VI concludes the paper.

II. CHANNEL MODELS

Shadowing is usually modeled by a lognormal distribution with standard deviation $\sigma$ and mean $\mu$; $\sigma$ defines the severity of shadowing. As the parameters of lognormal distribution are often given in decibels, the mappings $\sigma = \lambda \sigma_{dB}$ and $\mu = \lambda \mu_{dB}$ with $\lambda = \ln(10)/10$ can be utilized for the conversion. Besides, the small-scale multipath fading is often characterized by Nakagami distribution (occurs in Rayleigh-lognormal channel) [11], [12]. Then, the parameters of $\gamma$th gamma component are determined, the performance metrics are readily available [12] or can be easily derived.

B. SNR Distribution on the Relay Link

The instantaneous SNR on the relay link is modeled by a gamma-lognormal distribution (occurs in Nakagami-lognormal channel) [11], [12]. The parameters of $\gamma$th gamma component are expressed as [12]

$$\theta_i = \left(\frac{m_{RL}}{\gamma}\right)^{m_{RL}} w_i e^{-m_{RL} \sqrt{\sigma t_i} + \mu} \frac{\sqrt{\pi} I_{m_{RL}}(\gamma)}{\sqrt{\gamma} I_{m_{RL}+1}(\gamma)}$$

where $m_{RL}$ is the fading parameter of Nakagami distribution on the relay link (abbreviated by RL in this notation), $\gamma$ is a parameter of the MG distribution, and $t_i$ and $w_i$ are, respectively, abscissas and weight factors of $K$-th order Hermite integration.

C. SNR Distribution on the Access Link

An RN cell is typically characterized by small coverage area due to lower transmit power levels relative to the BSs [3], [6]. Accordingly, we assume that a direct LOS component along with many weak non-LOS (NLOS) scatter components exist on the propagation paths between an RN and an MT on the access link. Furthermore, the LOS component may be partially or completely blocked by surrounding objects, e.g., trees, which implies random shadowing [11]. Hence, we model the access link by Rician-lognormal distribution. Accordingly, we obtain the parameters of the $\gamma$th gamma component [9]:

$$\theta_i = \frac{1 + K}{\gamma} \left(\frac{m_{AL}}{m_{AL} + K}\right)^{m_{AL}} \left(\frac{(m_{AL})_{i-1}}{(m_{AL} + K)}\right)^{i-1} \frac{K(1 + K)}{(\Gamma(i))^2}$$

$$\beta_i = i, \quad \zeta_i = \frac{1 + K}{\gamma}, \quad K \triangleq \Omega/2b_0$$

where $0 \leq m_{AL} \leq \infty$ describes the severity of shadowing on the access link (abbreviated by AL in this notation), and $K$ is the Rician $K$ factor where $\Omega$ is the average power of the LOS component and $2b_0$ is the average power of the scatter component.

III. SYSTEM MODEL

A. Coarse Relay Site Planning Model

Cell planning and site selection tools are routinely used by operators to improve the system performance and to provide a
satisfactory service with minimal deployment expenditure. In this work, it is assumed that the original radio network planning has been done for a single-hop macrocell-only system. Then, RNs are introduced to improve the system performance.

Within the framework of RSP, an RN location is chosen from a set of possible locations. RSP takes into account the channel properties at different locations and considers their links’ qualities toward the serving BS in order to enhance the relay link quality. In particular, we assume that there are $M$ potential locations for RN deployment in cell $k$ out of which we select the best location in terms of downlink SINR considering shadowing only. In each location, RN is assumed to be served by a predefined BS solely. Then, the resultant SINR in the selected location is of the following form

$$\Upsilon_{m,k} = \max \{ \Upsilon_{m,k}^c : m = 1, 2, ..., M \},$$

(5)

where $\Upsilon_{m,k}^c$ is the SINR for the $m_{th}$ location in the $k_{th}$ cell. We note that while performing coarse RSP, we take into account only shadowing. Thus, the SINR at the selected location $\Upsilon_{m,k}$ can be different that of the actual SINR $\Upsilon_{\hat{m},k}$, which reflects the actual channel conditions impaired by both shadowing and multi-path fading. That is, coarse RSP is carried out based on $\Upsilon_{\hat{m},k}^c$; however, $\Upsilon_{\hat{m},k}$ is the experienced SINR during the operation.

### B. Multi-cellular Network Model

1) Network Layout and RSP Location Trellis: The considered network is represented by a regular hexagonal layout with 7 cells, where we look for a suitable location for a single RN in the $k_{th}$ cell assuming $M$ potential location candidates. Fig. 1 depicts the network layout along with the utilized RN location trellis. The RN location trellis models a practical scenario where the $M = 5$ candidate locations are localized in a target region. Moreover, $d_1$ denotes the distance between the serving BS and the midmost RN location. The outer candidate locations are at a distance of $d_2$ apart from the midmost candidate location.

2) Path-loss Model: The path loss, including shadowing, is given by

$$L_{m,k} = \alpha d_{m,k}^\beta 10^{\zeta_{m,k}/10}/G,$$

(6)

where $d_{m,k}$ the distance between $m_{th}$ potential relay location and $k_{th}$ BS, $k = 0, 1, 2, ..., K$. Further, $\alpha$ and $\beta$ are, respectively, a propagation constant and the path-loss exponent, together which define the distance dependent path-loss, $G$ is dimensionless and reflects the impact of antenna gain, which is assumed to be the same for each BS (isotropic antenna gain patterns are employed at BSs). Besides, $\zeta_{m,k}$ is a zero-mean Gaussian random variable (RV) that models the shadowing. RV $\zeta_{m,k}$ can be expressed as a sum of two independent zero-mean Gaussian RVs $\xi_{m,k}$ and $\eta_{m,k}$ with standard deviation of $\sigma_{db}$, where the former corresponds to the near field of the $m_{th}$ location and is the same for all BSs, and the latter variable is a BS-dependent variable, which is independent from one BS to the other [9]. Thus, we have

$$\zeta_{m,k} = \sqrt{\rho} \cdot \xi_{m} + \sqrt{1 - \rho} \cdot \eta_{m,k},$$

(7)

where $\rho$ is the correlation coefficient related to any pair of BSs. Accordingly, for $k \neq j$, we obtain

$$E(\zeta_{m,k}) = \rho \sigma_{db}^2, \quad E(\zeta_{m,k}^2) = \sigma_{db}^2, \quad E(\zeta_{m,k}) = 0.$$  (8)

That is, shadowing variables $\zeta_{m,k}$ and $\zeta_{m,j}$ are correlated. We note that in 3GPP studies the shadowing correlation coefficient of $\rho = 0.5$ between BSs is usually applied [6].

In accordance with the Gudmundson model [9], the correlation between shadowing samples at different locations in the $k_{th}$ cell is given by

$$\rho(\zeta_{m,k}, \zeta_{n,k}) = e^{-|d_{m,n}|/d_{cor}^2} \ln 2,$$

(9)

where $\zeta_{m,k}$ and $\zeta_{n,k}$ are the shadowing variables at locations $m$ and $n$, respectively, $d_{m,n}$ is the distance between the two locations, and $d_{cor}$ is the so-called de-correlation distance. The proposed value for $d_{cor}$ in, e.g., [13], is $20$ m. Then, shadowing correlation between potential RN positions with mutual distance of round $50$ m, is small and, thus, is neglected in the closed-form analysis. Due to the low correlation in shadowing between candidate locations, the correlation between SINR values is also low and can be ignored.

### IV. ANALYSIS OF COARSE RELAY SITE PLANNING

#### A. Derivation of the Actual Relay Link SINR

The actual SINR at the $m_{th}$ location considering composite fading/shadowing is of the following form [9]

$$\Upsilon_{m,k} = \frac{S_{m,k}^2 10^{X_{m,k}/10}}{P_N + \sum_{j \neq k} S_{m,j}^2 10^{X_{m,j}/10}},$$

(10)

where $S_{m,k}^2$ is the power envelope of the multi-path fading channel on the desired link between $k_{th}$ BS and $m_{th}$ location, which is modeled by Nakagami distribution, $S_{m,j}^2$ is the power envelope of the multi-path fading channel on the interfering link between $j_{th}$ BS and $m_{th}$ location, which is modeled by Rayleigh distribution, and $P_N$ denotes the thermal noise. Further, $X_{m,k} \sim N(\mu_{X_{m,k}}, \sigma_{db}^2)$ with $X_{m,k} = -\zeta_{m,k} + \mu_{X_{m,k}}$, and $X_{m,j} \sim N(\mu_{X_{m,j}}, \sigma_{db}^2)$ with $X_{m,j} = -\zeta_{m,j} + \mu_{X_{m,j}}$ are Gaussian RVs, where means $\mu_{X_{m,k}}$ and $\mu_{X_{m,j}}$ comprise BS transmit power levels, and...
distance dependent path losses defined in (6). For example, we have \( \mu_{X_{m,k}} = 10 \log_{10}(P_{Tx,k} G_O^{-1} d_{m,k}^{-\beta}) \) with \( P_{Tx,k} \) being the transmission power of the \( k \)th BS.

In order to derive an analytically tractable SINR expression, we need to tackle several difficulties. Concretely, an exact closed-form expression for the distribution of the sum of multiple lognormal and/or Suzuki RVs is not available. Moreover, the desired and interfering signals are mutually dependent due to shadowing, and there is a constant thermal noise term \( P_N \) in the denominator. Substituting (7) in (10) and after some algebraic manipulations detailed in [9], we obtain

\[
\tilde{\gamma}_{m,k} = \frac{S^2_{m,k} 10^{(\frac{1}{10} - \rho \eta_{m,k} + \mu_{X_{m,k}})}/10}}{P_N 10^{-\sqrt{10} \xi_{m}/10} + \sum_{j \neq k} S^2_{m,j} 10^{(\frac{1}{10} - \rho \eta_{m,j} + \mu_{X_{m,j}})}/10}.
\]

RVs in this re-formulated SINR expression are mutually independent. Moreover, the newly introduced RV \( P_N 10^{-\sqrt{10} \xi_{m}/10} \) follows a lognormal distribution with mean \( 10 \log_{10}(P_N) \) and standard deviation \( \sqrt{\rho \sigma_{dB}} \). The sum in the denominator consists of a multiple independent Suzuki RVs and a lognormal RV; therefore, it can be well approximated by a new lognormal RV consists of a multiple independent Suzuki RVs and a lognormal composite distribution which is characterized by an actual SINR distribution on the relay link follows a gamma-lognormal distribution defined in (11) where the parameter expressions are provided by (3) in which \( \gamma \) is set to one.

B. Derivation of the Relay Link SINR for Coarse RSP

When only shadowing is considered on the relay link, the SINR formulation in (11) can be re-written as

\[
\tilde{\gamma}_{m,k} = \frac{S^2_{m,k} 10^{(\frac{1}{10} - \rho \eta_{m,k} - Z + \mu_{X_{m,k}})}/10}}{\Delta_{m,k}} := \frac{S^2_{m,k} 10^{\Delta_{m,k}}}{10},
\]

where \( \Delta_{m,k} \) is a Gaussian RV with mean \( \mu_{X_{m,k}} - \mu_Z \) and standard deviation \( \sqrt{(1 - \rho) \sigma_{dB}^2 + \sigma_Z^2} \). Accordingly, the actual SINR distribution on the relay link follows a gamma-lognormal composite distribution which is characterized by (1)-(2), where the parameter expressions are provided by (3) in which \( \gamma \) is set to one.

C. Maximum Achievable Gains by Optimal RSP

In optimal RSP, the RN location is selected according to the gamma-lognormal composite distribution as provided in (12). Accordingly, when optimal RSP is carried out in the \( k \)th cell over \( M \) candidate locations, the CDF of the relay link SINR attains the following form

\[
F_{m,k}(\tilde{\gamma}) = \prod_{m=1}^{M} F_{m,k}(\tilde{\gamma}),
\]

where \( F_{m,k}(\tilde{\gamma}) \) is given by (2) following the discussion after (12). Hence, \( F_{m,k}(\tilde{\gamma}) \) provides the maximum achievable gains by optimal RSP. On the other hand, the coarse RSP assumes a pure shadowing channel (see Section IV-B) for the RN location selection as given in (5). Consequently, due to multi-path fading component in the selected location \( m \), the actual SINR may degrade and the associated distribution can deviate from \( F_{m,k}(\tilde{\gamma}) \).

D. Link and End-to-end Rate Derivations

In the \( k \)th cell and at the \( m \)th RN location, the relay link rate \( R_{r,m,k} \) is given in terms of the relay link SINR as

\[
R_{r,m,k} = \delta_r \cdot A_r \cdot \log_2 \left( 1 + B_r \cdot \tilde{\gamma}_{m,k} \right),
\]

where \( A_r \) and \( B_r \) are, respectively, the bandwidth and SINR efficiency factors, and \( \delta_r \) is the overhead scaling factor which accounts for, e.g., LTE overhead through reference symbols [9]. In case of performing RSP, we obtain the relay link rate \( R_{r,m,k} \) by utilizing the relay link SINR in the selected location, i.e., \( \tilde{\gamma}_{m,k} \). On the other hand, the access link instantaneous rate \( R_a \) is of the form

\[
R_a = \delta_a \cdot A_a \cdot \log_2 \left( 1 + B_a \cdot \gamma_a \right),
\]

where \( \gamma_a \) is the instantaneous SNR on the access link, and the parameters \( A_a, B_a \) and \( \delta_a \) may differ from \( A_r, B_r \) and \( \delta_r \).

The end-to-end rate is, then, given in terms of the rate on the two hops, where due to half-duplex operation, transmissions from BS to RN and from RN to MT are scheduled on different time slots. Time resources allocated for the relay link communication constitute \( \tau_r \) of the total system resources. Similarly, access link communication is scheduled on \( \tau_a \) of the total available resources, where resource normalization is given as \( \tau_r + \tau_a = 1 \). Subsequently, the end-to-end rate experienced by a single user served by RN in the \( k \)th cell and \( m \)th location is defined as the minimum of the user rate achieved on the relay and access links

\[
R_{r,m,k} = \min \{ \tau_r \cdot R_{r,m,k}, \tau_a \cdot R_a \},
\]

where rates on the relay and access links are scaled by the portion of resources allocated to each. When RSP is performed, the end-to-end rate \( R_{r,m,k} \) is formulated similarly considering \( R_{r,m,k} \) instead.
TABLE I
SYSTEM PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>General Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Carrier Frequency</td>
<td>2 GHz</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Frequency Planning</td>
<td>Reuse 1</td>
</tr>
<tr>
<td>Duplexing Scheme</td>
<td>Frequency Division Duplex (FDD)</td>
</tr>
<tr>
<td>ISD</td>
<td>500 m</td>
</tr>
<tr>
<td>RN Location Trellis</td>
<td>( d_1 = \text{ISD}/\sqrt{3} ) m, ( d_2 = 50 ) m</td>
</tr>
<tr>
<td>Bandwidth Efficiencies ((A_a, A_r))</td>
<td>0.88</td>
</tr>
<tr>
<td>SINR Efficiencies ((B_a, B_r))</td>
<td>0.8</td>
</tr>
<tr>
<td>Overhead Scaling Factors ((\delta_a, \delta_r))</td>
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</tr>
<tr>
<td>Thermal Noise Power Spectral Density (PSD)</td>
<td>-174 dBm/Hz</td>
</tr>
<tr>
<td><strong>BS Parameters</strong></td>
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</tr>
<tr>
<td>Transmit Power</td>
<td>46 dBm</td>
</tr>
<tr>
<td>Antenna Gain</td>
<td>14 dB</td>
</tr>
<tr>
<td>Antenna Configuration and Pattern</td>
<td>Tx-1, Omni-directional</td>
</tr>
<tr>
<td>Antenna Height</td>
<td>25 m (above rooftop)</td>
</tr>
<tr>
<td><strong>RN Parameters</strong></td>
<td></td>
</tr>
<tr>
<td>Antenna Gain</td>
<td>5 dB</td>
</tr>
<tr>
<td>Antenna Configuration and Pattern</td>
<td>Rx-1, Omni-directional</td>
</tr>
<tr>
<td>Antenna Height</td>
<td>5 m (below rooftop)</td>
</tr>
<tr>
<td>Noise Figure</td>
<td>5 dB</td>
</tr>
<tr>
<td><strong>Shadowing on the Relay Link</strong></td>
<td></td>
</tr>
<tr>
<td>De-correlation Distance ((d_{\text{cor}}))</td>
<td>20 m</td>
</tr>
<tr>
<td>Correlation Factor ((\rho))</td>
<td>0.5 between cells</td>
</tr>
<tr>
<td><strong>Relay Link Path-Loss</strong></td>
<td></td>
</tr>
<tr>
<td>Path-Loss Exponent ((\beta))</td>
<td>3.63</td>
</tr>
<tr>
<td>Propagation Constant ((\alpha_{\text{diff}}))</td>
<td>125.2 dB</td>
</tr>
</tbody>
</table>

V. PERFORMANCE EVALUATION

In this section, we evaluate the effect of coarse RSP on the relay link quality as well as on the end-to-end performance. Besides, we demonstrate the achievable gains relative to the performance bound. The simulations are conducted using MATLAB as the computational environment. Moreover, the simulation models follow the 3GPP guidelines given in [6].

In addition, we focus on coverage-oriented planning, i.e., RNs are positioned at the cell edge where users experience high interference and/or severe propagation losses toward the serving BS. In Table I, the system parameters are summarized in accordance with [6].

A. Relay Link SINR Distribution

The impact of RSP on the relay link SINR distribution is illustrated by CDF plots in Fig. 2 for two sets of channel parameters. These sets of channel parameters are:

1. \((m_{RL}; \sigma_{DB}) = (5.76; 6)\) which corresponds to a scenario with comparatively light fading, and
2. \((m_{RL}; \sigma_{DB}) = (1; 8)\) which corresponds to a scenario with severe fading.

It is noticed that coarse RSP provides high SINR gains especially at lower CDF percentiles in both scenarios. Moreover, it is observed that the gains via coarse RSP deviate less from the maximum achievable gains by optimal RSP in the first scenario particularly at high CDF percentiles. On the other hand, such deviation increases when the impact of multipath fading becomes more dominant, which is the case in the second scenario. Nevertheless, the observed gains through coarse RSP justify its impact in alleviating the effects of severe fading. We note that a typical \(\sigma_{DB}\) value of shadowing on the relay link is 6 dB, e.g., in LTE-Advanced standard [6]. Therefore, \(\sigma_{DB} = 6\) is adopted in the following.

B. Link and End-to-end Rates

Fig. 3 shows the CDFs of the access, relay, and the end-to-end link rates for \(M = 5\) (with RSP) and \(M = 1\) (no RSP). Two cases are considered for the channel conditions on the access link [9]:

1. Frequent heavy shadowing with average access link SNR of \(\tau_a = 10\) dB reflects relatively moderate channel conditions as shown in Fig. 3(a), while
2. Infrequent light shadowing with average access link SNR of \(\tau_a = 20\) dB corresponds to good channel conditions as shown in Fig. 3(b).

In both cases, we have \((m_{RL}; \sigma_{DB})=(5.76; 6)\) and the resource allocation parameters are set as \(\tau_r = \tau_a = 0.5\). It can be
seen that coarse RSP results in clear rate gain on the relay link relative to no RSP. The deviation from the maximum achievable SINR gains translates into deviation in rate gains on the relay link, as well. Nevertheless, the end-to-end rate depends on the capacities of both the relay and access links. In this regard, under moderate access link channel conditions, the end-to-end rate performance is limited mainly by the capacity of the access link; the CDF plots of end-to-end and access link rates almost overlap. Therefore, the end-to-end rate performance of coarse RSP is similar to that of optimal RSP at all CDF percentiles. On the other hand, under good access link channel conditions, the end-to-end rate through RSP is, by contrast, limited by the capacity of the relay link. In such a case, the deviation in end-to-end rate performance of coarse RSP from that of optimal RSP becomes notable at lower CDF percentiles. Yet, when performing coarse RSP, a significant gain in end-to-end rate relative to no RSP is still observed.

VI. CONCLUSION

In this paper, we have investigated coarse RSP as a practical technique to enhance the wireless relay link performance of RNs by exploiting their deployment flexibility. The RN location selection is carried out considering shadowing only, whereas the performance has been analyzed assuming composite fading/shadowing channels. The impact of co-channel interference on the relay link quality is also taken into account within the framework of multi-cellular wireless networks. Moreover, the achievable gains via coarse RSP are demonstrated and compared with the maximum achievable gains by optimal RSP.

Results show that performing coarse RSP still provides significant gains on the relay link SINR relative to no RSP, particularly boosting the low SINR regime. It is seen that coarse RSP deviates less from the maximum achievable gains when the multi-path fading is less severe. Achieved SINR gains on the relay link are shown to translate into clear improvements in end-to-end rates. Further, the deviation from the maximum achievable end-to-end rate gains becomes negligible when the access link is the bottleneck.

REFERENCES


