# Optimal Feedback Updating Period for Coordinated Multi-Point Transmission Schemes

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***Abstract*—This work concerns a limited feedback scheme taking into account the channel time-correlation during the feedback design in slow fading environment. We consider the Coordinated MultiPoint (COMP) system model with other-cell interference (OCI). We determine the feedback updating period which is triggered by the transmitters to enable the receiver to send the CSI. This updating period is function of the channel temporal correlation, available feedback bits, and the number of transmit antennas. An optimal updating period is derived such that it guarantees that the average signal-to-interference-and-noise ratio (SINR) is greater than or equal to that of the conventional feedback scheme that does not adapt its feedback period to the temporal correlation. The numerical results show performance improvement in terms of overall capacity and feedback overhead reduction when comparing the proposed adaptive scheme with conventional feedback approaches.**

## I. Introduction

The availability of channel state information (CSI) at the transmitters highly impact the performance of any wireless system. In frequency division duplex (FDD) modes, CSI is obtained from the User Equipement (UE) through feedback links. The CoMP technologies were introduced by the Third Generation Partnership Project (3GPP) in Long Term Evolution-Advanced (LTE-A) as a feature of Release11. A transmission mode 10 (TM10) was specified in [2] for CoMP implementation and when a UE is configurated in TM10 by higher layers, can feed back multiple periodic or aperiodic CSI reports corresponding to one or more CSI processes per serving cell on UL links to his activated serving cell(s). As consequence, the UL overhead increase due to CSI measurement related to multiple points. In this paper, we consider a single-user joint-transmission (SU-JT) scheme composed of cooperating and interfering cells. We study a feedback scheme that takes channel temporal correlation into account in order to reduce the average feedback overhead while improving the capacity performance compared to a conventional feedback scheme. Reducing feedback overhead saves bandwidth in both uplink channel and backhaul connections. In this scope, our object is to find how often a receiver should feed back CSI to the transmitters. Then, a feedback updating period $n$ means that the receiver feeds back to the BSs updated CSI every $n$ channel slots. The feedback updating period is originally calculated for point-to-point MIMO systems in [3], and for large MIMO systems in [4].

The paper is organized as follows. In section II, we give the correlated channel model and the system model. In section III, we calculate the feedback updating period for the SU-JT scheme. Simulation results showing feedback reduction and capacity improvement are in section V.

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## II. Channel and System Models

### A. Channel Model

Consider a COMP system model and a single-user joint transmission cooperative scheme. The system consists of $B$ base stations which (BSs) jointly serve one single-antenna UE. The transmitting power of the $i$th BS is $P_i$. Consider also $M$ interfering cells with transmitting power $P_j$ per BS. All the transmitters at the BSs are equipped with $n_t$ antennas each. The UE's received signal at each channel slot $m$ is

$$y_m = \sum_{i=1}^{B} \sqrt{P_i} \boldsymbol{h}_{m,i}^H \boldsymbol{v}_{m,i} s_{m,i} + \sum_{j=1}^{M} \sqrt{P_j} \boldsymbol{h}_{m,j}^H \boldsymbol{v}_{m,j} s_{m,j} + w_m \quad (1)$$

$w_m \in \mathbb{C}$ is the additive white Gaussian noise $\sim \mathcal{CN}(0, \sigma_w^2{=}1)$. $\boldsymbol{h}_{m,i}$ $(\boldsymbol{h}_{m,j}) \in \mathbb{C}^{n_t \times 1}$ is the channel vector between the $i$th ($j$th interfering) BS and the UE and it is spacially uncorrelated flat fading channel. $\boldsymbol{v}_{m,i}$ $(\boldsymbol{v}_{m,j}) \in \mathbb{C}^{n_t \times 1}$ is the beamforming vector at the $i$th ($j$th) BS. $s_{m,i}$ $(s_{m,i})$ $\in \mathbb{C}$ is the transmitted signal by the $i$th ($j$th) BS satisfying $\mathbb{E}\left[|s_{m,i}|^2\right] = \mathbb{E}\left[|s_{m,j}|^2\right] = 1$. Without loss of generality, we assume that all BSs have equal transmitting powers, i.e., $P_i = P$ and $P_j = P$. Then, the received signal-to-interference-noise ratio SINR at channel slot $m$ is given by

$$\mathsf{SINR}_m = \frac{P \sum_{i=1}^{B} \left| \boldsymbol{h}_{m,i}^H \boldsymbol{v}_{m,i} \right|^2}{\sigma_w^2 + P \sum_{j=1}^{M} \left| \boldsymbol{h}_{m,j}^H \boldsymbol{v}_{m,j} \right|^2} \quad (2)$$

In the following section, we calculate the feedback updating period for this system model.

The channel time evolution of the channel is modeled by a first-order Gauss-Markov process

$$\boldsymbol{h}_m = \alpha \boldsymbol{h}_{m-1} + \sqrt{1-\alpha^2} \boldsymbol{g}_m, \quad (3)$$

where $\boldsymbol{g}_m \mathbb{C}^{n_t \times 1}$ is the innovation process having independent and identically distributed (i.i.d.) entries distributed according to $\mathcal{CN}(0,1)$. The initial state $\boldsymbol{h}_0$ is independent of $\boldsymbol{g}_{m+1}$ for all $m$. The time correlation coefficient $\alpha \in [0,1]$ represents the correlation between the elements $h_{m,i}$ and $h_{m+1,i}$, and is the same for all the elements of $\boldsymbol{h}_m$. Throughout the paper, $\tilde{\boldsymbol{h}}$ denotes the unit vector direction of $\boldsymbol{h}$, $\tilde{\boldsymbol{h}} = \boldsymbol{h}/\|\boldsymbol{h}\|$, and $\boldsymbol{h}^H$ denotes the Hermitian conjugated transposition operation.

### *B. Feedback Frameworks*

In a conventional random vector quantization limited feedback framework, *Scheme A*, the receiver feeds back to the transmitter the index of the best beamforming vector for each channel slot $m$. The beamforming vector is searched over a codebook $\mathcal{V} = \{\boldsymbol{v}_1, \ldots, \boldsymbol{v}_{N_1}\}$, where $N_1 = 2^{B_1}$, by maximizing the instantaneous beamforming gain

$$\boldsymbol{v}_m = \arg\max_{\boldsymbol{v}\in\mathcal{V}} \left|\tilde{\boldsymbol{h}}_m^H \boldsymbol{v}\right|^2 \tag{4}$$

At UE, the effective SNR is $\left|\boldsymbol{h}_m^H \boldsymbol{v}_m\right|^2$, and the feedback updating period is 1.

In this work, we consider the feedback scheme proposed in [3], *Scheme B*. The idea is to exploit the channel temporal correlation to optimize the feedback updating period. That is, the UE feeds back the beamforming vector index every $n$, $(n > 1)$, channel slots. For this scheme, the random codebook is $\mathcal{F} = \{\boldsymbol{f}_1, \ldots, \boldsymbol{f}_{N_2}\}$, where $N_2 = 2^{B_2}$. More specifically, for each measured channel, at $m = 0$ the UE feeds back the index of the beamforming vector by solving

$$\boldsymbol{f}_0 = \arg\max_{\boldsymbol{f}\in\mathcal{F}} \left|\tilde{\boldsymbol{h}}_0^H \boldsymbol{f}\right|^2 \tag{5}$$

The beamformer $\boldsymbol{f}_0$ is used at the transmitter for the channel slots $m = 0$ to $m = n-1$. At $m = n$, the UE sends back the next beamforming vector index by solving

$$\boldsymbol{f}_n = \arg\max_{\boldsymbol{f}\in\mathcal{F}} \left|\tilde{\boldsymbol{h}}_n^H \boldsymbol{f}\right|^2 \tag{6}$$

The beamformer $\boldsymbol{f}_n$ is used during the next $n$ channel slots. In the case when the channel is in a state of deep fading for a long time and the UE needs JT to improve his SINR, the average feedback overhead is reduced by controlling the feedback updating period $n$. The updating period $n$ and the size of codebook $\mathcal{F}$ have a great impact on the performance of scheme B. Thus, for fixed $N_1$, $N_2$, $n_t$, and $\alpha$, and given $N_2 > N_1$, we want to know whether it is possible to find the maximum feedback updating period $n$ ensuring that the performance of Scheme B is better or at least equal to that of Scheme A. As a consequence, a reduction of the feedback overhead and an enhancement of the performance of scheme B are possible in slow fading regime. This reveals the importance of taking into account the channel temporal correlation into the CoMP feedback design.

## III. Feedback Updating Period: Considering Other-Cells Interference

We use the average SINR loss at the channel slot $m$ to measure the performance of Scheme B, and to determine the updating period $n$. The average SINR loss at the $m$th channel instance between Scheme B and Scheme A is,

$$D_m^{\mathsf{SINR}} = \mathbb{E}\left[\frac{P\sum_{i=1}^{B}\left|\boldsymbol{h}_{m,i}^H \boldsymbol{f}_{0,i}\right|^2}{1+P\sum_{j=1}^{M}\left|\boldsymbol{h}_{m,j}^H \boldsymbol{v}_{m,j}\right|^2} - \frac{P\sum_{i=1}^{B}\left|\boldsymbol{h}_{m,i}^H \boldsymbol{v}_{m,i}\right|^2}{1+P\sum_{j=1}^{M}\left|\boldsymbol{h}_{m,j}^H \boldsymbol{v}_{m,j}\right|^2}\right] \tag{7}$$

For the sake of simplifying the calculation, we consider that the interfering cells are applying the Scheme A constantly. However, in the simulations, this hypothesis is omitted, and the results hold which proves that this assumption does not have great impact on the derived optimal feedback period. Moreover, the interference terms are independent between them, and independent of the signal terms.

Determining the feedback updating period requires closed-form expressions for the average beamforming gains of Scheme A and Scheme B in Eq.7. We address this issue in the following.

### *A. Average* SINR *of Scheme A*

*Lemma 1:* The average beamforming gain of Scheme A with other-cell interference and random vector quantization scheme at the channel instance $m$ $(m \geq 0)$ is given by

$$\mathsf{SINR}_{\mathsf{avg}}^{\mathsf{A}} = P \times n_t \times \mathcal{SA}(N_1), \tag{8}$$

where

$$\mathcal{SA}(N_1) = 1 - \sum_{l=0}^{N_1}\sum_{j_1,j_2,\ldots,j_J}\binom{l}{j_1,j_2,\ldots,j_J}\frac{N_1!(-1)^l}{l!(N_1-l)!} \times \left[\frac{1}{(Pn_t)^{Mn_t}(Mn_t-1)!}\right]^l \mathcal{I}_1(\nu,\xi,l,j_1,\ldots,j_J) \tag{9}$$

and $\mathcal{I}_1(\nu,\xi,l,j_1,\ldots,j_J)$ is given by

$$\mathcal{I}_1(\nu,\xi,l,j_1,\ldots,j_J) = \int_0^1 \mathcal{A}_1(\nu)^{j_1}\mathcal{A}_2(\nu)^{j_2}\ldots\mathcal{A}_J(\nu)^{j_J}d\nu \tag{10}$$

and

$$\mathcal{A}_{(i,k)\triangleq j}(\nu) = \frac{(Mn_t+k-1)!\nu^i e^{-\nu}}{k!(i-k)!\left(\nu+\frac{1}{Pn_t}\right)^{Mn_t+k}} \tag{11}$$

*Proof:* See Appendix A for a sketch of the proof. ■

### *B. Average* SINR *of Scheme B*

*Lemma 2:* The average beamforming gain of Scheme B with OCI and random vector quantization scheme at the channel instance $m$ $(m \geq 0)$ can be put in the form

$$\mathsf{SINR}_{\mathsf{avg,m}}^{\mathsf{B}} = P\|\boldsymbol{h}_m\|^2\left[\alpha^{2m}\mathbb{E}\left[\frac{\sum_{i=1}^{B}\left|\tilde{\boldsymbol{h}}_{0,i}^H\boldsymbol{f}_{0,i}\right|^2}{1+P\sum_{j=1}^{M}\left|\boldsymbol{h}_{m,j}^H\boldsymbol{v}_{m,j}\right|^2}\right] +(1-\alpha^2)\sum_{k=0}^{m-1}\alpha^{2k}\mathbb{E}\left[\frac{\sum_{i=1}^{B}\left|\tilde{\boldsymbol{g}}_{m-k,i}^H\boldsymbol{f}_{0,i}\right|^2}{1+P\sum_{j=1}^{M}\left|\boldsymbol{h}_{m,j}^H\boldsymbol{v}_{m,j}\right|^2}\right]\right] \tag{12}$$

*Proof:* The average SINR is determined by steps for $m = 0, 1, \ldots$. Trivially at $m = 0$, the average SINR of Scheme B is given by

$$\mathsf{SINR}^{\mathsf{B}}_{\mathsf{avg},0} = \mathbb{E}\left[\frac{P\left\|\boldsymbol{h}_m\right\|^2 \sum_{i=1}^{B}\left|\tilde{\boldsymbol{h}}_{0,i}^{H}\boldsymbol{f}_{0,i}\right|^2}{1 + P\sum_{j=1}^{M}\left|\boldsymbol{h}_{0,j}^{H}\boldsymbol{v}_{0,j}\right|^2}\right] \tag{13}$$

We consider that the average power of the interference is constant across the time, $\mathbb{E}\left[\sum_{j=1}^{M}\left|\boldsymbol{h}_{m,j}^{H}\boldsymbol{v}_{m,j}\right|^2\right]$, $\forall m$. At $m = 1$, we have

$$\mathsf{SINR}^{\mathsf{B}}_{\mathsf{avg},1}\Big/\left[Pn_t\right] = \mathbb{E}\left[\frac{\sum_{i=1}^{B}\left|\tilde{\boldsymbol{h}}_{1,i}^{H}\boldsymbol{f}_{0,i}\right|^2}{1 + P\sum_{j=1}^{M}\left|\boldsymbol{h}_{1,j}^{H}\boldsymbol{v}_{1,j}\right|^2}\right] = \tag{14}$$

$$\alpha^2\mathbb{E}\left[\frac{\sum_{i=1}^{B}\left|\tilde{\boldsymbol{h}}_{0,i}^{H}\boldsymbol{f}_{0,i}\right|^2}{1 + P\sum_{j=1}^{M}\left|\boldsymbol{h}_{1,j}^{H}\boldsymbol{v}_{1,j}\right|^2}\right] + (1-\alpha^2)\mathbb{E}\left[\frac{\sum_{i=1}^{B}\left|\tilde{\boldsymbol{g}}_{1,i}^{H}\boldsymbol{f}_{0,i}\right|^2}{1 + P\sum_{j=1}^{M}\left|\boldsymbol{h}_{1,j}^{H}\boldsymbol{v}_{1,j}\right|^2}\right],$$

where the equality follows from the facts that $\tilde{\boldsymbol{h}}_{0,i}^{H}$ and $\tilde{\boldsymbol{g}}_{1,i}$ are independent and the expectations of the cross-terms are zeros. Proceeding in a similar way for $m = 2$,

$$\begin{aligned}\mathsf{SINR}^{\mathsf{B}}_{\mathsf{avg},2}\Big/\left[Pn_t\right] = \;& \alpha^4\mathbb{E}\left[\frac{\sum_{i=1}^{B}\left|\tilde{\boldsymbol{h}}_{0,i}^{H}\boldsymbol{f}_{0,i}\right|^2}{1 + P\sum_{j=1}^{M}\left|\boldsymbol{h}_{2,j}^{H}\boldsymbol{v}_{2,j}\right|^2}\right] \\ & + (1-\alpha^2)\sum_{k=0}^{1}\alpha^{2k}\mathbb{E}\left[\frac{\sum_{i=1}^{B}\left|\tilde{\boldsymbol{g}}_{2-k,i}^{H}\boldsymbol{f}_{0,i}\right|^2}{1 + P\sum_{j=1}^{M}\left|\boldsymbol{h}_{2,j}^{H}\boldsymbol{v}_{2,j}\right|^2}\right]\end{aligned} \tag{15}$$

The generalization for $m$ is straightforward, and the result in Eq.12 is proved. Note that we have used the independence between the direction and the amplitude of $\boldsymbol{h}_m$ and $\boldsymbol{g}_{m-k}$, and used the fact that the amplitudes satisfy $\mathbb{E}\left[\left\|\boldsymbol{h}_m\right\|^2\right] = \mathbb{E}\left[\left\|\boldsymbol{g}_{m-k}\right\|^2\right] = n_t$. ■

Now that we have put the average beamforming gain of Scheme B in the form of Eq.12, we need to compute each of the right-hand side (r.h.s.) terms.

*1) Average of the First Term in Eq.12:* The average of the first term of the r.h.s. of Eq.12 is given by Eq.8 while replacing $N_2$ by $N_1$,

$$\mathbb{E}\left[\frac{\sum_{i=1}^{B}\left|\tilde{\boldsymbol{h}}_{0,i}^{H}\boldsymbol{f}_{0,i}\right|^2}{1 + P\sum_{j=1}^{M}\left|\boldsymbol{h}_{m,j}^{H}\boldsymbol{v}_{m,j}\right|^2}\right] = \mathcal{SA}\left(N_2\right)$$

*2) Average of the Second Term in Eq.12:* We follow the same steps as in Lemma.1 to calculate the average of the second term of Eq.12. We formulate the result in the following.

*Lemma 3:* Consider

$$x \triangleq \frac{\sum_{i=1}^{B}\left|\tilde{\boldsymbol{g}}_{m-k}^{\star i}\boldsymbol{f}_0\right|^2}{1 + P\sum_{j=1}^{M}\left|\boldsymbol{h}_m^{\star j}\boldsymbol{v}_m\right|^2} := \frac{w}{1+y}, \tag{16}$$

$y$ is a chi-squared random variable, $y \sim \mathcal{X}^2_{2Mn_t}$. $w$ is a normal random variable, $w \sim \mathcal{N}\left(\mu_w, \sigma_w^2\right)$. The average of $x$ is

$$\begin{aligned}\mathbb{E}\left[x\right] = \;& \frac{1}{2} - \frac{1}{\sqrt{\pi}}\sum_{j=0}^{\infty}\sum_{k=0}^{2j+1}\sum_{l=0}^{k}\frac{(-1)^{3j+k+1}(2j)!\mu_w^{2j+1-k}}{\left(\sqrt{2\sigma_w^2}\right)^{2j+1} j!l!(2j+1-k)!} \\ & \frac{(Mn_t+l-1)!\left(Pn_t\right)^l}{(k-l)!(Mn_t-1)!(k+1)} \\ \overset{(a)}{=} \;& \mathcal{ASB}\end{aligned} \tag{17}$$

Step $(a)$ is for ease of notations. Note that $\mathcal{ASB}$ is independent of $N_1$, $N_2$, and $m$.

*Proof:* See Appendix B for a sketch of the proof. ■

At this point, the average SINR of Scheme B is obtained by plugging Eq.8 applied to $N_2$ and Eq.17 into Eq.12. We obtain

$$\begin{aligned}\mathbb{E}\left[\mathsf{SINR}^{\mathsf{B}}_m/P\left\|\boldsymbol{h}_m\right\|^2\right] &= \alpha^{2m}\mathcal{SA}\left(N_2\right) + (1-\alpha^2)\sum_{k=0}^{m-1}\alpha^{2k}\mathcal{ASB} \\ &= \alpha^{2m}\mathcal{SA}\left(N_2\right) + (1-\alpha^{2m})\mathcal{ASB} \\ &= \alpha^{2m}\left(\mathcal{SA}\left(N_2\right) - \mathcal{ASB}\right) + \mathcal{ASB}\end{aligned} \tag{18}$$

### *C. The Optimal Updating Period*

Plugging Eq.8 and Eq.18 into the average effective SINR loss in Eq.7 gives

$$D_m^{\mathsf{SINR}} = Pn_t\left(\alpha^{2m}\left(\mathcal{SA}\left(N_2\right) - \mathcal{ASB}\right) + \mathcal{ASB} - \mathcal{SA}\left(N_1\right)\right) \tag{19}$$

The optimization problem of finding the updating period of Scheme B guaranteeing greater SINR gain than Scheme A is formulated by finding the optimal channel index $m^{\mathsf{SINR}}$ verifying

$$\begin{aligned} m^{\mathsf{SINR}} \;=\; & \arg\min_{m\geq 0} D_m^{\mathsf{SINR}} \\ & s.t.\; D_m^{\mathsf{SINR}} \geq 0. \end{aligned} \tag{20}$$

Solving Eq.20 is equivalent to find maximum $m$ such that $D_m^{\mathsf{SINR}} \geq 0$. Then, the feedback updating period is $n^{\mathsf{SINR}} = m^{\mathsf{SINR}}+1$. Manipulating the condition on $D_m^{\mathsf{SINR}}$ yields a bound on $m$, and the solution of Eq.20 is

$$m^{\mathsf{SINR}} = \left\lfloor \log_{\alpha^2}\left(\frac{\mathcal{SA}\left(N_1\right) - \mathcal{ASB}}{\mathcal{SA}\left(N_2\right) - \mathcal{ASB}}\right)\right\rfloor_+. \tag{21}$$

$\lfloor\rfloor_+$ is the flooring function of the nearest non-negative integer.The updating period $n^{\mathsf{SINR}}$ must satisfy

$$n^{\mathsf{SINR}} \leq \left\lfloor \log_{\alpha^2}\left(\frac{\mathcal{SA}\left(N_1\right) - \mathcal{ASB}}{\mathcal{SA}\left(N_2\right) - \mathcal{ASB}}\right)\right\rfloor_+ + 1. \tag{22}$$

$n^{\mathsf{SINR}}$ is function of the channel correlation $\alpha$, available feedback resources, and the number of transmit antennas.

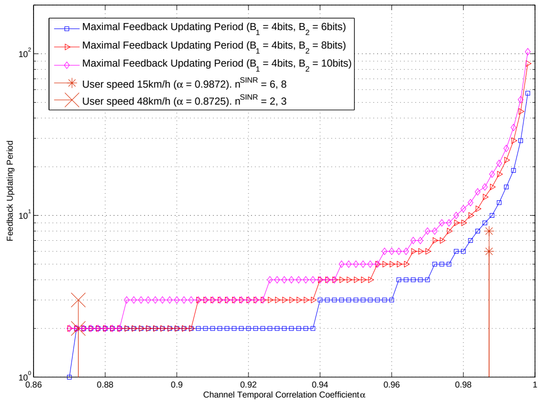


Fig. 1. Maximum Feedback Updating Period of Scheme B for $B_1 = 4$bits and $B_2 = 6, 8, 10$bits when considering other-cell interference.

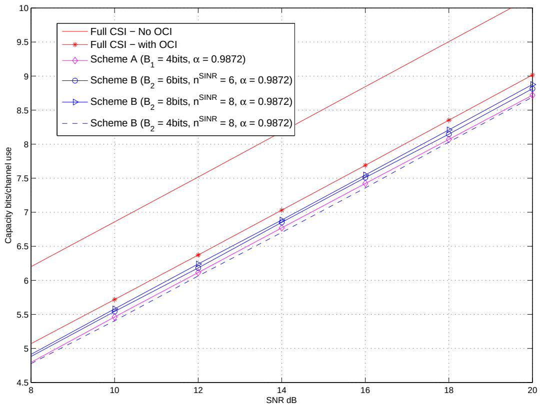


Fig. 2. Throughput performance of Scheme A (4bits/channel use) and Scheme B (1bits/channel use) when considering other-cell interference. User speed 15km/h.

## IV. Numercial Results

We evaluate the capacity performance of Scheme B and Scheme A in slow fading environment via Monte Carlo simulations. We consider 3 cooperating BSs and 9 neighbouring interfering BSs, all applying the same feedback reuse scheme.

The time correlation coefficient of the channel $\alpha$ is generated by Jake's model according to $\alpha = J_0\left(2\pi f_D T\right)$, where $J_0()$ is the 0th order Bessel function of the first kind, $f_D$ is the maximum Doppler frequency, and $T$ is the symbol duration. We consider scenarios and parameters akin to 3GPP LTE-A specifications. The BSs are equipped with $n_t = 4$ antennas each. For the delay model, we use a carrier frequency $f_c = 2.6$Ghz as in frequently deployed in LTE systems, and a time transmission interval $T = 1$ms. The LTE-A supports terminal's speed up to 360km/h.

In Fig.1, we plot the upper-bound on the maximal feedback updating period given in Eq.21 for $B_1 = 4$bits, and $B_2 = 6, 8, 10$ as a function of the channel time correlation coefficient $\alpha$. The plotted values of $\alpha$ correspond to a mobile speed ranging from $5km/h$ $(0.9986)$ to $48km/h$ $(0.8725)$. As $\alpha$ approaches 1, the feedback updating period increases drastically as the channel does not change and the mobile doesn't move. However, for $\alpha < 0.84$, when mobile speed exceeds 54km/h, the optimal feedback updating period is 1. In this figure we highlight the cases corresponding to $\alpha = 0.9872$ and $\alpha = 0.8725$, with the updating periods of $(6; 8)$ and $(2; 3)$ respectively. We can conclude from Fig.1 that whenever $B_2$ increases, i.e. the feedback is finer, the feedback updating period can be extended as long as it guarantees an improved performance over Scheme A.

In Fig.2, we plot the achievable capacity with full CSI with and without OCI. The Scheme A is considered with $B_1 = 4$bits, and the Scheme B with $B_2 = 6$ bits and $B_2 = 8$bits, respectively. The feedback updating period are $n^{\mathsf{SNR}} = 6$, and $n^{\mathsf{SNR}} = 8$, respectively. These values of $n^{\mathsf{SINR}}$ satisfy the upper-bound in Fig.1. We consider that the user is at equal distance to the cooperating BSs, and is moving at a speed of 15km/h corresponding to $\alpha = 0.9872$. The results show that by controlling the updating feedback period, Scheme B can present an improved capacity gain over Scheme A while reducing the feedback overhead at the same time. Indeed, Scheme A uses 4bits/channel use, whereas Scheme B uses 1bits/channel use. We also plot the performance of Scheme B with $B_2 = 4$bits, and $n^{\mathsf{SNR}} = 8$. This shows us the gain obtained by increasing $B_2$ from 4 to 8 in Scheme B.

In Fig.3 another scenario is considered, where UE's speed of $48km/h$ $(\alpha = 0.8725)$. We use a feedback updating period of 2 and 3, respectively. For an updating period of 2, Scheme B can perform better than Scheme A. However, for $n^{\mathsf{SINR}} = 3$, we get worse performance. Even for medium speed the reduced feedback scheme can outperfom a conventional scheme but more often we need to update the channel information.

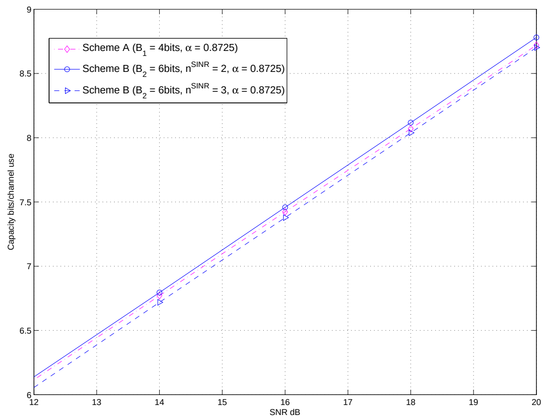


Fig. 3. Throughput performance of Scheme A (4bits/channel use) and Scheme B (3bits/channel use and 2bits/channel use) when considering other-cell interference. User speed 48km/h.

## V. Conclusion

We have treated the optimization problem of finding the optimal feedback updating period in a limited feedback beamforming system. We have considered a single-user joint-transmission COMP scheme with interference from non-cooperating cells. We have characterized bounds on the feedback updating period depending on the channel time-correlation coefficient, the available feedback bits, and the number of transmit antennas. The proposed feedback framework showed improved capacity performance with reduced average feedback overhead than a conventional feedback scheme based on block-by-block feedback. The feedback overhead reduction is crucial in the overall performance of COMP systems as in some architecture CSI is exchanged between BSs through limited backhaul links. These results reveal the necessity of taking temporal correlation into account to the feedback design. For a future work, we will investigate the multi-user-CoMP scheme, and issues related to decentralized scheduler and BSs clustering.

## APPENDIX

*A: Proof of Lemma 1*

The instantaneous $\mathsf{SINR}_m^{\mathsf{A}}$ is given by

$$\mathsf{SINR}_m^{\mathsf{A}} = \max_{v\in\mathcal{V}} xP\left\|\boldsymbol{h}_m\right\|^2 \tag{23}$$

by defining

$$\nu = \max_{v\in\mathcal{V}} x \tag{24}$$

$$x \triangleq \frac{\sum_{i=1}^{B}\left|\tilde{\boldsymbol{h}}_i^H\boldsymbol{v}\right|^2}{1+P\sum_{j=1}^{M}\left|\boldsymbol{h}_j^H\boldsymbol{f}\right|^2} := \frac{z}{1+y} \tag{25}$$

$z$ is the sum of $B$ independent chi-squared random variables, i.e. $z \sim \mathcal{X}^2_{2Bn_t}$, and $y$ is the sum of $M$ independent chi-squared random variables, i.e. $y \sim \mathcal{X}^2_{2Mn_t}$. The amplitude of $\boldsymbol{h}_m$ satisfies $\mathbb{E}\left[\left\|\boldsymbol{h}_m\right\|^2\right] = n_t$. We need to compute $\mathbb{E}\left[\nu\right]$.

Given the pdf of $y$ in $[Eq.3, [5]]$, and the expression of $F_{Z|Y}(z)$ in $[Eq.32, [6]]$, we first derive the cumulative distribution function (cdf) of the random variable $x$, for $x\in[0,1]$,

$$\begin{aligned}F_X(x) &= P\left(\frac{z}{1+y}\le x\right) &(26)\\ &= \int_0^{\infty} F_{Z|Y}\left(x\left(1+y\right)\right)p_Y(y)dy &(27)\\ &= 1-\sum_{i=0}^{Bn_t-1}\sum_{k=0}^{i}\frac{M^{Mn_t}(Mn_t+k-1)!}{k!(i-k)!(Mn_t-1)!(Pn_t)^{Mn_t}}\\ &\quad\times\frac{e^{-x/B}x^i}{\left(x+\frac{M}{BPn_t}\right)^{Mn_t+k}}\times\frac{1}{B^{Mnt+k-1}} &(28)\end{aligned}$$

Second, we follow the steps in [7] to find the cdf of $\nu$ using a random $N_1$ unit vector beamforming codebook. The cdf of $\nu$ is the same as that of $x$ raised to the $N_1$-th power,

$$\begin{aligned}F_\nu(\nu) &= \left[F_X(x)|_{x=\nu}\right]^{N_1} &(29)\\ &= \sum_{l=0}^{N_1}\frac{N_1!(-1)^l}{l!(N_1-l)!}\left[\frac{e^{-\nu}}{(Pn_t)^{Mn_t}(Mn_t-1)!}\right]^l\\ &\quad\times\sum_{j_1,j_2,\ldots,j_J}\binom{l}{j_1,j_2,\ldots,j_J}\\ &\quad\times\mathcal{A}_1(\nu)^{j_1}\mathcal{A}_2(\nu)^{j_2}\ldots\mathcal{A}_J(\nu)^{j_J}. &(30)\end{aligned}$$

We obtain the last result using the multinomial theorem and adequate change of variables. Third, the expectation of $\nu$ is derived as follows

$$\begin{aligned}\mathbb{E}\left[\nu\right] &= \int_0^1 \nu p_\nu(\nu)d\nu &(31)\\ &= \int_0^1\left(1-F_\nu(\nu)\right)d\nu &(32)\\ &= \mathcal{SA}\left(N_1\right) &(33)\end{aligned}$$

*B: Proof of Lemma 3* We follow the same steps as in Lemma.1 to calculate the average of the second term of Eq.12. $w$ is the sum of independent beta distributed variables of the same parameters; $\mathsf{Beta}(\alpha_i,\beta_i)$ with $\alpha=\alpha_i=1$, and $\beta=\beta_i=n_t-1, \forall i$. Using the central limit theorem, we approximate $w$ by a normal distribution with mean $\mu_w = B/n_t$ and variance $\sigma_w^2 = B(n_t-1)/(n_t^2(n_t+1))$. The cumulative distribution function of $w$ is

$$F_W(w) = \frac{1}{2}\left[1+\mathsf{erf}\left(\frac{w-\mu_w}{\sqrt{2\sigma_w^2}}\right)\right] = \frac{2}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n z^{(2n+1)}}{n!(2n+1)}, \tag{34}$$

$\mathsf{erf}(z)$ is the error function, and the second equality is given by the Taylor series expansion of $\mathsf{erf}(z)$.

First, we derive the cumulative distribution function of the variable $x\in[0,1]$ as follows.

$$\begin{aligned}F_X(x) &= P\left(\frac{w}{1+y}\le x\right) &(35)\\ &= \int_0^{\infty}F_{W|Y}\left(x\left(1+y\right)\right)p_Y(y)dy &(36)\\ &= \frac{1}{2}+\frac{1}{\sqrt{\pi}}\sum_{j=0}^{\infty}\sum_{k=0}^{2j+1}\sum_{l=0}^{k}\frac{(-1)^{3j+k+1}(2j)!}{\left(\sqrt{2\sigma_w^2}\right)^{2j+1}j!l!(k-l)!}\\ &\quad\times\frac{\mu_w^{2j+1-k}(Mn_t+l-1)!}{(2j+1-k)!(Mn_t-1)!}\left(\frac{1}{Pn_t}\right)^{-l}x^k &(37)\end{aligned}$$

This restult is obtained using the pdf of $y$ in $[Eq.3, [5]]$, and the binomial expansion.

Second, the expectation of $x$ is given by,

$$\mathbb{E}\left[x\right] = \int_0^1(1-F_X(x))dx = \mathcal{ASB} \tag{38}$$

## REFERENCES


[1] D. Gesbert, S. Hanly, H. Huang, S. S. Shitz, O. Simeone, and W. Yu, "Multi-Cell MIMO Cooperative Networks: A New Look at Interference," *IEEE J. on Sel. Areas In Comm.*, vol.28, no.9, pp. 1-29, december 2010.

[2] 3rd Genration Partnership Project (3GPP). TS 36.213 V11.0.0 (2012-09) "Evolved-Universal Terrestril Radio Access (E-UTRA); Physical layer procedures."

[3] T. Kim, D. J. Love, and B. Clerckx, "Leveraging Temporal correlation for limited feedback multiple antennas systems," *IEEE Procedings ICASSP*, Dallas, USA, march 2010.

[4] W. Santipach, and K. Mamat, "Optimal Feedback Interval for Temporally-Correlated Multiantenna Channel," *IEEE Proceedings Globecom*, USA, 2011.

[5] M. S. Alouini, and A. J. Goldsmith, "Capacity of Rayleigh Fading Channels under Different Adaptive Transmission and Diversity-Combining Techniques," *IEEE Trans. on Veh. Tech.*, vol.48, no.4, pp 1165 - 1181, 1999.

[6] K. K. Mukkavilli, A. Sabharoual, E. Erkip, and B. Aazhang, "On Beamforming with Finite-Rate Feedback in Multi-Antenna Sysytems," *IEEE Trans. on Inf. Theroy*, vol.49, no.10, pp 2562 - 2579, October 2003.

[7] C. K. Au-Yeung, and D. J. Love, "On the Performance of Random Vector Quantization Limited Feedback Beamforming in a MISO System," *IEEE Trans. on Wire. Comm*, vol.6, no.2, pp 458 - 462, February 2007.

[8] T. Sellke, "On the Fitting of Pearson Curves to Sums of Independent Random Variables," Dep. of Statistics, Stanford University, May 1983.