Outage Analysis of Various Cooperative Strategies for the Multiple Access Multiple Relay Channel

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***Abstract*—In this paper, we derive the individual and common symmetric ϵ-outage achievable rate of various Multiple Access Multiple Relay Channel (MAMRC) cooperation schemes. Common to all the schemes: (1) The sources are independent and want to communicate with a single destination with the help of multiple relays; (2) Each relay is half-duplex and apply a Selective Decode and Forward (SDF) relaying strategy, i.e., it forwards only a deterministic function of the error-free decoded messages. The schemes differ in the assignment of the available channel uses to the sources and the relays, varying from the less efficient orthogonal assignment where each source and relay is given non-overlapping channel uses, to the most efficient one where each source and relay can transmit on all the available channel uses. In this paper, no Channel State Information at the Transmitter (CSIT) is assumed and all links are independent and subject to slow fading and additive white Gaussian noise.**

I. Introduction

The Multiple Access Multiple Relay Channel (MAMRC) consists of M independent users (sources) attempt to communicate with a common destination with the help of L independent relays. The capacity region of Multiple Access Channel (MAC) ($L = 0$) and the techniques to exploit it are well known [1]. These techniques depend, in the general case, on Non-Orthogonal Multiple Access (NOMA). The idea of using relays to enlarge the MAC capacity region has motivated many research efforts during the past decade. For $L \geq 1$ the capacity region, hence the optimal relaying strategy, is not known [2].

Most practical physical layer Joint Network Channel Coding (JNCC) (or distributed code) designs proposed in the literature rely on Orthogonal Multiple Access (OMA) since it greatly facilitates their decoding (no interference management at the receiver side). The main purpose of this paper is to evaluate theoretically the performance loss due to this orthogonality assumption. By removing gradually the constraint of orthogonality between the links, we formulate the symmetric individual and common ϵ-outage achievable rate of various MAMRC cooperation schemes. The symmetric individual ϵ-outage achievable rate R_ϵ^{ind} is defined as the highest transmission rate of each source such that the probability of any source to be in outage is less or equal to ϵ. The symmetric common ϵ-outage achievable rate R_ϵ^{com} is defined as the highest transmission rate of each source such that the probability of a common outage event, which is defined as the event of having at least one source in outage, is less or equal to ϵ. We assume that no channel state information is available at the transmitter and all links are independent, and follow a slow Rayleigh fading distribution.

The chosen relaying strategy is called Selective Decode and Forward (SDF), i.e., the relay try to decode the messages of the sources then, based on a Cyclic Redundancy Check (CRC), forwards a deterministic function of the correctly decoded ones. It was first analyzed in a Separate Network Channel Coding (SNCC) framework from an information outage perspective in [3] and further extended to JNCC for MARC in [4], [5]. By SNCC we mean that the channel coding is exclusively dedicated to turn the radio links into block erasure channels. We restrict our outage analysis to this particular relaying strategy in a JNCC framework because (1) It prevents the error propagation from the relay to the destination; (2) It reduces the energy consumption at the relays and limits the interference within the network (the relay is always helpful when it cooperates); (3) It breaks down the MAMRC into parallel MACs whose common and individual outage rate regions are perfectly known. We are well aware that other competing strategies exist in the literature, see, e.g., [6], but theirs comparisons with SDF exceed the scope of this paper and is left for further studies.

Due to the half-duplex constraint of the L relays, the total number of channel uses N available to transmit a source message (packet) is divided into two time phases, namely, αN for the listening phase of the relay and $\bar{\alpha} N$ for its transmission phase where $\alpha \in]0, 1]$, and $\bar{\alpha} = 1 - \alpha$. In our baseline cooperation scheme Orthogonal MAMRC (OMAMRC) each source transmits only during the first phase on $\alpha N/M$ channel uses, while each relay transmits only during the second phase on $\bar{\alpha} N/L$ channel uses. By removing gradually the constraint of orthogonality between the radio links, we obtained 3 alternative cooperation schemes. The Semi-Orthogonal MAMRC (SOMAMRC) type I is defined as follows. The sources transmit during the first phase and keep silent while the relays transmit. The sources and the relays both use NOMA. The SOMAMRC type II is defined as follows. The sources in the first phase use OMA, i.e., each one of them is allocated $\alpha N/M$ channel uses. The relays and

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the sources are allowed to jointly transmit during the second phase using NOMA. This cooperation scheme is particularly interesting since it results in a low complexity receiver at the relay. To the best of the authors' knowledge, it has never been investigated before. Finally, the Non-Orthogonal MAMRC (NOMAMRC) is defined as follows. The sources are allowed to transmit during the first phase and during the second phase with the relays. All links are non orthogonal using NOMA. Its derivation is detailed in [7] for the single transmit and receive antenna case. This last cooperation scheme yields an upper bound on the other ones. Fig. 1 summarizes all the aforementioned cooperation schemes.

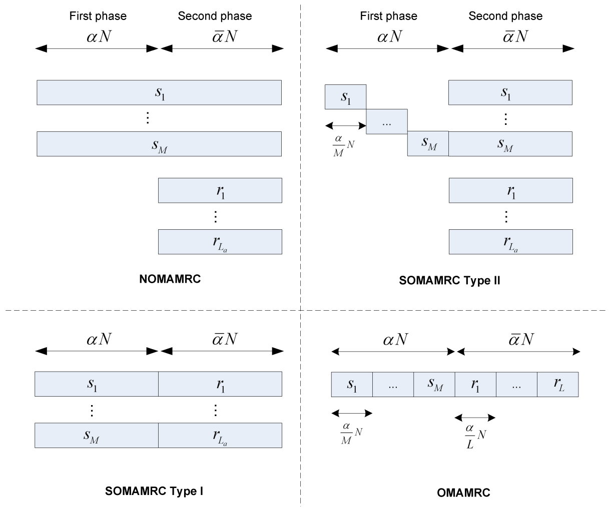

Fig. 1. Radio resources splitting in SOMAMRC Type I, SOMAMRC Type II, OMAMARC and NOMAMRC

In this paper, we focus on SOMAMRC type II analysis since the other cooperation schemes' outage derivation follow the same line. In Section II, we introduce the system model of SOMAMRC Type II. Section III is devoted to the outage analysis of SOMAMRC type II. Numerical results are presented in Section IV comparing all the considered cooperation schemes. Some conclusions are drawn in Section V.

Notation In the sequel, we use boldface letters to denote vector and matrices. Matrices are represented by capital letters. Let $\mathbf{A}$ be a matrix with i^{th} row $\mathbf{a}^i$ and j^{th} column $\mathbf{a}_j$, entry (i, j) is denoted $a_{i,j}$ or equivalently $[\mathbf{A}]_{i,j}$. The n-square identity matrix is denoted $\mathbf{I}_n$. $\mathbf{x} \sim p(\mathbf{x})$ means that the random vector $\mathbf{x}$ follows the probability distribution function (pdf) $p(\mathbf{x})$. $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that $\mathbf{x}$ is a circularly symmetric complex Gaussian random vector with mean $\mathbb{E}(\mathbf{x}) = \boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$. Calligraphic upper case letters are used to denote integer finite sets of the form $\mathcal{S} = \{s_1, \ldots, s_{|\mathcal{S}|}\}$ where $|\mathcal{S}|$ is the cardinality of the set $\mathcal{S}$. The empty set is denoted by $\emptyset$. Let $\mathbf{X}_{\mathcal{S}}$ denote the matrix $[\mathbf{x}_{s_1}, \ldots, \mathbf{x}_{s_{|\mathcal{S}|}}]$. Similarly, we define the block matrix $\underline{\mathbf{X}}_{\mathcal{S}}$ as $[\mathbf{X}_{s_1}, \ldots, \mathbf{X}_{s_{|\mathcal{S}|}}]$. Given a condition C we define $\mathbf{1}_{\{C\}}$ as an indicator function, i.e., $\mathbf{1}_{\{C\}} = 1$ if C is true and $\mathbf{1}_{\{C\}} = 0$ otherwise. $I(\mathbf{x}; \mathbf{y})$ denotes the mutual information between two random vectors $\mathbf{x}, \mathbf{y}$.

II. SOMAMRC Type II System Model

Let us consider a set of statistically independent sources $\mathcal{S} = \{s_1, \ldots, s_M\}$, each source $s \in \mathcal{S}$ wants to communicate its message $\mathbf{u}_s \in \mathbb{F}_2^K$ of K information bits to a common destination d with the help of a set of relays $\mathcal{R} = \{r_1, \ldots, r_L\}$. We assume each source s is equipped with n_s transmit antenna, each relay r is equipped with n_r transmit antenna and m_r receive antenna, and the destination is equipped with m_d receive antenna. We focus on the symmetric case where $n_s = n_r = n, \forall s \in \mathcal{S}$, and $\forall r \in \mathcal{R}$. Let N be the total number of available complex dimensions. The listening phase of each relay lasts $N_1 = \alpha N$ channel uses, and its transmission phase $N_2 = \overline{\alpha} N$, where $\overline{\alpha} = 1 - \alpha$, with $\alpha \in]0, 1]$. Note that we always assume that the number of channel uses defined in this paper are integers. During the first phase, each source $s \in \mathcal{S}$ is given a specific time slot to transmit the first part of its codeword denoted by $\mathbf{X}_s^{(1)} \in \mathbb{C}^{n \times \frac{N_1}{M}}$ (which is itself a codeword). The received signals at the relay $r \in \mathcal{R}$ and the destination d at the time slot associated to user s can be written as

$$\mathbf{y}_{r,k}^{(s,1)} = \sqrt{\frac{\gamma_{sr}}{n}} \mathbf{H}_{sr} \mathbf{x}_{s,k}^{(1)} + \mathbf{n}_{r,k}^{(1)}, \tag{1}$$

and

$$\mathbf{y}_{d,k}^{(s,1)} = \sqrt{\frac{\gamma_{sd}}{n}} \mathbf{H}_{sd} \mathbf{x}_{s,k}^{(1)} + \mathbf{n}_{d,k}^{(1)} \tag{2}$$

where $k = 1, \ldots, N_1/M$, respectively. During the second phase, each source s continues to transmit the second part of its codeword denoted by $\mathbf{X}_s^{(2)} \in \mathbb{C}^{n \times N_2}$. Let $\mathcal{S}_r \subseteq \mathcal{S}$ denote the maximum set of sources' messages that the relay r can decode error free. If the set $\mathcal{S}_r$ is empty then r remains silent in this phase. Otherwise, the relay r transmits a modulated sequence $\mathbf{X}_r^{(2)} \in \mathbb{C}^{n \times N_2}$. The sequence $\mathbf{X}_r^{(2)}$ is chosen such that $[\underline{\mathbf{X}}_{\mathcal{S}_r}, \mathbf{X}_r^{(2)}]$ is a (modulated) codeword on the messages $\mathbf{u}_{\mathcal{S}_r}$. The received signal at the destination can be written as

$$\mathbf{y}_{d,k}^{(2)} = \sum_{s \in \mathcal{S}} \sqrt{\frac{\gamma_{sd}}{n}} \mathbf{H}_{sd} \mathbf{x}_{s,k}^{(2)} + \sum_{r \in \mathcal{R}_a} \sqrt{\frac{\gamma_{rd}}{n}} \mathbf{H}_{rd} \mathbf{x}_{r,k}^{(2)} + \mathbf{n}_{d,k}^{(2)} \tag{3}$$

where $k = 1, 2, \ldots, N_2$ and $\mathcal{R}_a$ denotes the set of active relays. In (1), (2), and (3), we assume that the additive noises $\mathbf{n}_{r,k}^{(1)}$, $\mathbf{n}_{d,k}^{(1)}$, and $\mathbf{n}_{d,k}^{(2)}$ are i.i.d. circularly symmetric complex Gaussian random vectors with covariance equal to identity matrix, the entries of the channels matrices $\mathbf{H}_{ab}$, where $a \in \{s, r\}$, and $b \in \{r, d\}$, are i.i.d, zero-mean circularly symmetric complex Gaussians, i.e., $[\mathbf{H}_{ab}]_{i,j} \sim \mathcal{CN}(0, 1)$. The channel matrices $\mathbf{H}_{ab}$ stay constant over N channel uses after which they switch to an independent value. γ_{ab} is the Signal-to Noise Ratio (SNR) related to a per received antenna (assuming that all sources of interference are perfectly canceled). No channel state information at the transmitter and perfect channel state information at the receiver are assumed. Hence, using the conjecture in [8], the sources and the relays (when they transmit) split the power equally over all the transmit antennas and send an independent data stream over each transmitting antenna, i.e., $\mathbb{E}(\mathbf{x}_s \mathbf{x}_s^{\dagger}) = \mathbb{E}(\mathbf{x}_r \mathbf{x}_r^{\dagger}) = \mathbf{I}_n$.

III. Information-Theoretic analysis

In this Section, we assume classically that $N \to \infty$ and that all the transmitted sequences are i.i.d such that the Asymptotic Equipartition Property (AEP) holds [1, Chapter 3]. In SOMAMRC type II, the sources in the first phase use OMA while the relays and the sources in the second phase use NOMA. Based on the fact that the relays apply SDF relaying strategy we can derive the rate regions, corresponding to the different outage events, at the relays in the first phase then, conditional to the obtained outage events at the relays, we can obtain the rate regions corresponding to the outage events at the destination. Hence, conditional on the channel states $\mathbf{H} = [\underline{\mathbf{H}}_{\mathcal{S}r_1}, \ldots, \underline{\mathbf{H}}_{\mathcal{S}r_L}, \underline{\mathbf{H}}_{\mathcal{S}d}, \underline{\mathbf{H}}_{\mathcal{R}d}]$ we can formulate perfectly the individual and the common outage events. Let us define the mutually independent input random vectors $\mathbf{x}_s^{(1)} \sim p(\mathbf{x}_s^{(1)})$, $\mathbf{x}_s^{(2)} \sim p(\mathbf{x}_s^{(2)})$, and $\mathbf{x}_r^{(2)} \sim p(\mathbf{x}_r^{(2)})$, $\forall s \in \mathcal{S}$, and $\forall r \in \mathcal{R}$ and the independent output random vectors $\mathbf{y}_d^{(s,1)}$, $\mathbf{y}_d^{(2)}$, and $\mathbf{y}_r^{(s,1)}$, whose associated channel transition conditional pdfs follow the ones associated to (2), (3), and (1), respectively.

A. Slow fading MAC Outage analysis

The analysis in this section are general and are not related to previously described SOMAMRC Type II scheme, but its results are extremely important not only for SOMAMRC Type II but also for all the other schemes described in the introduction. Let us consider the $|\mathcal{S}|$-users MAC defined as

$$\mathbf{y}_{d,k} = \sum_{s \in \mathcal{S}} \sqrt{\frac{\gamma_{sd}}{n}} \mathbf{H}_{sd} \mathbf{x}_{s,k} + \mathbf{n}_{d,k} \tag{4}$$

where $k = 1, \ldots, N$ (with $N \to \infty$) , $\mathbf{n}_{d,k}$ are i.i.d circularly symmetric complex Gaussian random vectors. The symmetric achievable rate $R = R_s\ \forall s \in \mathcal{S}$ of the previously defined MAC [1], [9] is given by

$$R_{\mathcal{U}} \leq I(\mathbf{X}_{\mathcal{U}}; \mathbf{y}_d | \mathbf{X}_{\mathcal{U}^c}, \underline{\mathbf{H}}_{\mathcal{S}d}) \quad \text{for all} \quad \mathcal{U} \subseteq \mathcal{S} \tag{5}$$

where $\mathcal{U}^c = \mathcal{S} \backslash \mathcal{U}$, $R_{\mathcal{U}} = |\mathcal{U}| R$ and given the input distribution $\prod_{s \in \mathcal{U}} p(\mathbf{x}_s)$. When equal orthogonal multiple access (OMA) is used, i.e., each source is given an equal amount of channel uses, the previous symmetric achievable rate is reduced and the rate constraints in (5) become

$$I(\mathbf{X}_{\mathcal{U}}; \mathbf{y}_d | \mathbf{X}_{\mathcal{U}^c}, \underline{\mathbf{H}}_{\mathcal{S}d}) = \frac{1}{M} \sum_{s \in \mathcal{U}} I(\mathbf{x}_s; \mathbf{y}_d^{(s)} | \underline{\mathbf{H}}_{\mathcal{S}d}) \tag{6}$$

where

$$\mathbf{y}_{d,k}^{(s)} = \sqrt{\frac{\gamma_{sd}}{n}} \mathbf{H}_{sd} \mathbf{x}_{s,k} + \mathbf{n}_{d,k}.$$

For the sake of notation simplicity, we remove the channel state from the outage event definitions and mutual information expressions in the following. Let $\mathcal{O}_{d,s}$ denote the symmetric individual outage event of source s, and $\mathcal{E}_{d,\mathcal{S}}$ denote the symmetric common outage event at the destination d of the $|\mathcal{S}|$-users MAC. Using (5), (6) this event could be expressed as

$$\mathcal{E}_{d,\mathcal{S}} = \{R_{\mathcal{U}} > I(\mathbf{X}_{\mathcal{U}}; \mathbf{y}_d | \mathbf{X}_{\mathcal{U}^c}) \quad \text{for some} \quad \mathcal{U} \subseteq \mathcal{S}\}, \tag{7}$$

or equivalently,

$$\mathcal{E}_{d,\mathcal{S}} = \bigcup_{\mathcal{U} \subseteq \mathcal{S}} \mathcal{F}_{d,\mathcal{S}}(\mathcal{U}) \tag{8}$$

where $\mathcal{F}_{d,\mathcal{S}}(\mathcal{U})$ is defined as the outage event of sources $\mathcal{U}$ assuming the messages of $\mathcal{U}^c = \mathcal{S} \setminus \mathcal{U}$ are perfectly known. This event can be expressed as

$$\mathcal{F}_{d,\mathcal{S}}(\mathcal{U}) = \{R_{\mathcal{U}} > I(\mathbf{X}_{\mathcal{U}}; \mathbf{y}_d | \mathbf{X}_{\mathcal{U}^c})\}. \tag{9}$$

When any $\mathcal{F}_{d,\mathcal{S}}(\mathcal{U})$ holds, the destination d can not decode all the messages of $\mathcal{U}$ knowing perfectly the messages of $\mathcal{U}^c$. In this case, a symmetric common outage of sources $\mathcal{S}$ is declared. The fact that $\mathcal{E}_{d,\mathcal{S}}$ holds does not mean that the destination can not decode error free the messages of a subset of $\mathcal{S}$. Excluding $\mathcal{S}$ itself and the empty subset, we can define $2^M - 2$ reduced MACs as follows

Definition (1) *A $|\mathcal{I}^c|$-users reduced MAC is a MAC with a subset of sources $\mathcal{I}^c$ of the original MAC, considering the complement of this subset $\mathcal{I} = \mathcal{S} \backslash \mathcal{I}^c$ as interference.*

Definition (2) *An expanded MAC of a $|\mathcal{I}^c|$-users reduced MAC is a MAC that contains at least the $\mathcal{I}^c$ original sources plus one.*

Let $\mathcal{E}_{d,\mathcal{I}^c}$ denote the common outage event of the $|\mathcal{I}^c|$-users reduced MAC. We can express this event by

$$\mathcal{E}_{d,\mathcal{I}^c} = \bigcup_{\mathcal{U} \subseteq \mathcal{I}^c} \mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U}) \tag{10}$$

where

$$\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U}) = \{R_{\mathcal{U}} > I(\mathbf{X}_{\mathcal{U}}; \mathbf{y}_d | \mathbf{X}_{\mathcal{U}_c})\}. \tag{11}$$

This equation is very similar to (8). However, in $\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U})$ the set of sources $\mathcal{I}$ are considered as interference, i.e, only the sources belonging to $\mathcal{U}_c = \mathcal{I}^c \backslash \mathcal{U}$ are supposed to be perfectly known.

Proposition (1) *The source s is in outage iff the $|\mathcal{S}|$-users MAC and all the reduced MAC containing s are in outage.*

$$\mathcal{O}_{d,s} = \bigcap_{\mathcal{I} \subset \mathcal{S}: s \in \mathcal{I}^c} \mathcal{E}_{d,\mathcal{I}^c} \tag{12}$$

$proof$: The sufficient part: if all the reduced MAC containing s are in outage then the message of s can not be decoded (error free) by any possible mean, thus, the source s is in outage. The necessary part: if the source s is in outage and one reduced MAC including this source is not in outage, it means that the destination can jointly decode the sources of this reduced MAC, as a result, the destination can decode the message of user s which contradicts the statement that s is in outage. ■

In some cases, i.e., at the relays, we are interested in finding $\mathcal{S}_d$ the maximum set of sources that the destination can decode.

Proposition (2) *The sufficient and necessary condition for a set of sources to be $\mathcal{S}_d$ is (i) the $|\mathcal{S}_d|$-users reduced MAC is not in outage and (ii) all the expanded MAC of this $|\mathcal{S}_d|$-users reduced MAC are in outage.*

proof : First $|\mathcal{S}_d|$-user reduced MAC is not in outage is a sufficient and necessary condition for the set $\mathcal{S}_d$ to be jointly decodable, now to make sure that it is the maximum set we need that the sufficient and necessary condition for other sets with higher cardinality to be invalid which is guarantee by the second condition of the proposition. ■

Finally, the source s symmetric individual and common outage probability can be expressed as

$$P_{out,ind} = \int_{\mathbf{H}} \mathbf{1}_{\{\mathcal{O}_{d,s}(\mathbf{H})\}} p(\mathbf{H}) d(\mathbf{H}) \tag{13}$$

and

$$P_{out,com} = \int_{\mathbf{H}} \mathbf{1}_{\{\mathcal{E}_{d,\mathcal{S}}(\mathbf{H})\}} p(\mathbf{H}) d(\mathbf{H}), \tag{14}$$

respectively, where $\mathbf{H} = \underline{\mathbf{H}}_{\mathcal{S}d}$. Note that the outage probability turns out to be a tight upper bound on achievable average Block Error Rate (BLER) even for a finite number of channel uses (around several hundreds) [10]. Indeed, practical distributed coding schemes and related receiver architectures in [4] were able to approach within a couple of dB the outage probability of SOMARC type I for $L = 1$. We define the symmetric individual and common ϵ-outage achievable rate by

$$R_\epsilon^{ind} = sup\{R : P_{out,ind} \leq \epsilon\} \tag{15}$$

and

$$R_\epsilon^{com} = sup\{R : P_{out,com} \leq \epsilon\}, \tag{16}$$

respectively.

B. SOMAMRC Type II Outage Analysis

1) Relay Outage Analysis: During the first transmission phase, we have a $|\mathcal{S}|$-users MAC at each relay $r \in \mathcal{R}$ where OMA is used. Thus, the definitions and analyses of the previous section remain valid hereafter, we simply have to replace the index d by r. Since each relay $r \in \mathcal{R}$ relies on the SDF strategy, we are interested in finding the maximum set of sources $\mathcal{S}_r$ with whose the relay r can cooperate with. The set $\mathcal{S}_r$ determines the cooperation mode of the relay r. As a result, we have 2^M cooperation modes for each relay and we have 2^{ML} cooperation modes considering the relays altogether.

2) Destination Outage Analysis: In this Section, we assume the best decoding scheme at the destination, i.e., it relies on joint network channel decoding. Since the source-to-destination and the source-and-relay-to-destination MACs associated with each successive transmission phase are non interfering, they can be viewed as one $|\mathcal{S}|$-users MAC made of two parallel MACs. As a result, the individual outage event of source s is given by (12) and the common outage event is given by (10). To compute $I_{\mathcal{I}^c}(\mathcal{U})$ in $\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U})$, we need to partition the relays into three sets (i) if $r \in \mathcal{R}_I$ the relay signal $\mathbf{x}_r^{(2)}$ is interference, (ii) if $r \in \mathcal{R}_k$ the relay signal $\mathbf{x}_r^{(2)}$ is perfectly known, (iii) if $r \in \mathcal{R}_u$ the relay signal $\mathbf{x}_r^{(2)}$ is jointly decodable with the signals $\mathbf{x}_\mathcal{U}$. The definitions of the sets $\mathcal{R}_I$, $\mathcal{R}_k$, $\mathcal{R}_u$ are given below where $\mathcal{U}_c = \mathcal{I}^c \setminus \mathcal{U}$

Definition (3) *$\mathcal{R}_I = \{r \in \mathcal{R}_a : \mathcal{I} \cap \mathcal{S}_r \neq \emptyset\}$ the set of relays whose signals are interference in $\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U})$*

Definition (4) *$\mathcal{R}_k = \{r \in \mathcal{R}_a : \mathcal{S}_r \subseteq \mathcal{U}_c\}$ the set of relays whose signals are known without decoding in $\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U})$.*

Definition (5) *$\mathcal{R}_u = \mathcal{R}_a \setminus \{\mathcal{R}_k \cup \mathcal{R}_I\}$ the set of relays whose signals are to be jointly decoded with the sources belonging to $\mathcal{U}$ in $\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U})$*

Using the above partitioning, we can express $\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U})$ as

$$\begin{aligned}\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U}) = \{R_\mathcal{U} > \frac{\alpha}{M} \sum_{s \in \mathcal{U}} I(\mathbf{x}_s^{(1)}; \mathbf{y}_d^{(s,1)}) + \\ \bar{\alpha} I(\mathbf{X}_\mathcal{U}^{(2)}, \mathbf{X}_{\mathcal{R}_u}^{(2)}; \mathbf{y}_d^{(2)} | \mathbf{X}_{\mathcal{U}_c}^{(2)}, \mathbf{X}_{\mathcal{R}_k}^{(2)})\}.\end{aligned} \tag{17}$$

We consider the following example for illustration.

Example (1) *Consider a 3-users 3-relays SOMAMRC Type II and take the following scenario for the relays cooperation $\mathcal{S}_{r_1} = \{s_1\}$, $\mathcal{S}_{r_2} = \{s_1, s_2\}$, and $\mathcal{S}_{r_3} = \{s_2, s_3\}$. Now let us consider, during our search to find if source s_2 is in outage or not, the event $\mathcal{F}_{d,\{s_1,s_2\}}(\{s_2\})$ which is the outage event of source s_2 assuming that the signal s_1 is known and s_3 is interference. In this case, the signal of r_3 is interference, r_1 is known, and r_2 is a part of the codeword corresponding to source s_2 (conditional to the knowledge of the signal of s_1). So we can write*

$$\begin{aligned}\mathcal{F}_{d,\{s_1,s_2\}}(\{s_2\}) = \{R > \frac{\alpha}{3} I(\mathbf{x}_{s_2}^{(1)}; \mathbf{y}_d^{(s_2,1)}) + \\ \bar{\alpha} I(\mathbf{x}_{s_2}^{(2)}, \mathbf{x}_{r_2}^{(2)}; \mathbf{y}_d^{(2)} | \mathbf{x}_{s_1}^{(2)}, \mathbf{x}_{r_1}^{(2)})\}.\end{aligned}$$

For Gaussian input distributions the instantaneous mutual information expressions in $\mathcal{F}_{d,\mathcal{I}^c}(\mathcal{U})$ can be expressed as

$$I(\mathbf{x}_s^{(1)}; \mathbf{y}_d^{(s,1)}) = \log \left| \mathbf{I}_{m_d} + \frac{\gamma_{sd}}{n} \mathbf{H}_{sd} \mathbf{H}_{sd}^\dagger \right|, \tag{18}$$

$$\begin{aligned}&I(\mathbf{X}_\mathcal{U}^{(2)}, \mathbf{X}_{\mathcal{R}_u}^{(2)}; \mathbf{y}_d^{(2)} | \mathbf{X}_{\mathcal{U}_c}^{(2)}, \mathbf{X}_{\mathcal{R}_k}^{(2)}) = \\ &\log \frac{\left| \mathbf{I}_{m_d} + \sum_{s \in \mathcal{U} \cup \mathcal{I}} \frac{\gamma_{sd}}{n} \mathbf{H}_{sd} \mathbf{H}_{sd}^\dagger + \sum_{r \in \mathcal{R}_u \cup \mathcal{R}_I} \frac{\gamma_{rd}}{n} \mathbf{H}_{rd} \mathbf{H}_{rd}^\dagger \right|}{\left| \mathbf{I}_{m_d} + \sum_{s \in \mathcal{I}} \frac{\gamma_{sd}}{n} \mathbf{H}_{sd} \mathbf{H}_{sd}^\dagger + \sum_{r \in \mathcal{R}_I} \frac{\gamma_{rd}}{n} \mathbf{H}_{rd} \mathbf{H}_{rd}^\dagger \right|}.\end{aligned} \tag{19}$$

IV. Numerical Results

In this Section, we consider only Gaussian i.i.d inputs. Due to space limitation we will show the results of the symmetric individual ϵ-outage achievable rate R_ϵ^{ind}, which is computed, for $\epsilon = 0.01$, and $\alpha = 2/3$. We fix the number of users to $M = 4$. There are an infinity of SNR configurations $\gamma_{r,d}$, $\gamma_{s,r}$, $\gamma_{s,d}$. By taking a few arbitrary configurations as examples, we want to illustrate the behaviors of the different schemes. We compute also R_ϵ^{ind} of M-users MAC considering no relay in

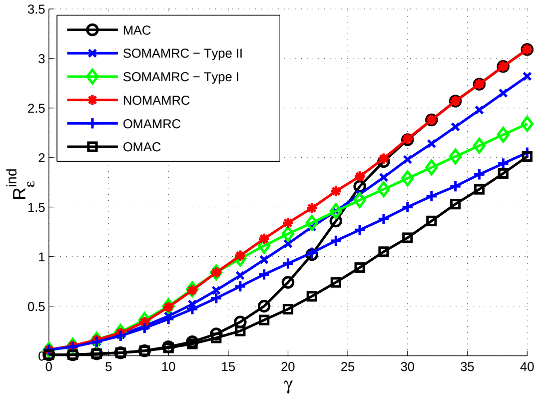

Fig. 2. $R_\epsilon^{ind}(\gamma)$ where $M = 4$, $L = 1$, and $n = m_d = m_r = 1$.

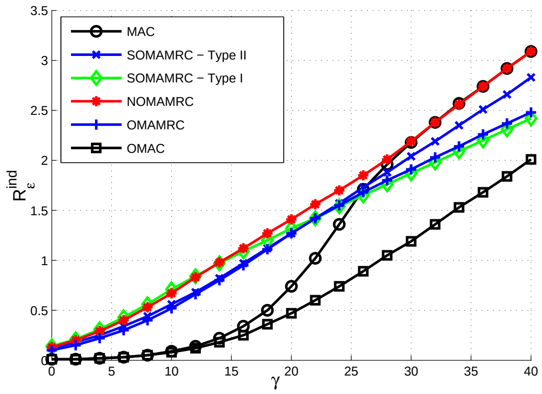

Fig. 3. $R_\epsilon^{ind}(\gamma)$ where $M = 4$, $L = 2$, and $n = m_d = m_r = 1$.

the system. As a result, the N channel uses are assigned to the sources to transmit either using OMA or NOMA.

In the first set of simulations, we assume all the sources, the relays, and the destination are equipped with single receive or/and single transmit antenna, i.e., $n = m_d = m_r = 1$, For SOMAMRC Type I we chose $\gamma_{r,d} = \gamma_{s,r} = \gamma_{s,d} = \gamma$. We consider this choice in order to validate our results with the ones of [4], [7]. To fairly compare the performance of the different schemes we will assume the same energy budget per source (per available dimensions). Hence, for OMAMRC we have $\gamma_{r,d} = \gamma$, $\gamma_{s,r} = M * \gamma$, and $\gamma_{s,d} = M * \gamma$, for SOMAMRC Type II we have $\gamma_{r,d} = \gamma$, $\gamma_{s,r} = M * \gamma$, and $\gamma_{s,d} = \frac{\alpha}{\alpha/M+\bar{\alpha}}\gamma$, and for NOMAMRC we have $\gamma_{r,d} = \gamma$, $\gamma_{s,r} = \gamma$, and $\gamma_{s,d} = \alpha\gamma$. Fig. 2, and 3 show R_ϵ^{ind} of the four schemes when the system contains 1, and 2 relays, respectively. In general, we notice that

- the cooperative schemes have the same achievable rates at low SNR,
- as expected, the NOMAMRC has the best achievable rate over all SNR, and OMAMRC has the worst achievable rate over all the SNR,
- the SOMAMRC Type I is as good as NOMAMRC at moderate SNR which is not the case at high SNR where SOMAMRC Type II becomes better,
- at high SNR, R_ϵ^{ind} of the NOMAMRC scheme converges to the one of the MAC(NOMA), In [7] it was proven, for $n = m_d = m_r = 1$, that the relays can not help any more, and the probability of the relay to be not active goes to one; all the other schemes have less achievable rates than the MAC(NOMA) since the sources in these schemes do not make use of all the available channel uses and the probability of the relays to be passive is high.

In the second set of simulations, we increase the reliability of the source-to-relay channels by adding 10 dB with respect to the previous simulations. Fig. 4 shows the R_ϵ^{ind} for all the schemes where $L = 2$. As we can see we still have the same performance as the first set of simulations. but at moderate SNR the performance of SOMAMRC Type I becomes more clear. In the third set of simulation, we choose to put 4 receive

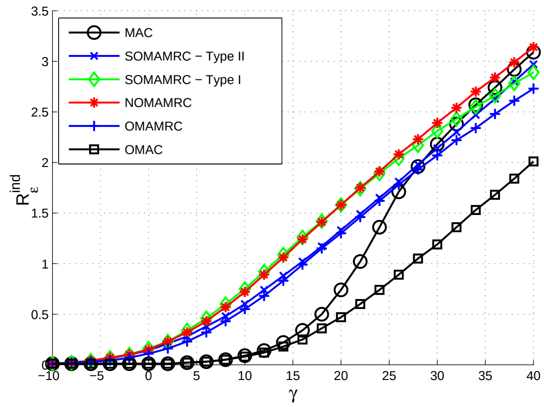

Fig. 4. $R_\epsilon^{ind}(\gamma)$ where $M = 4$, $L = 2$, and $n = m_d = m_r = 1$.

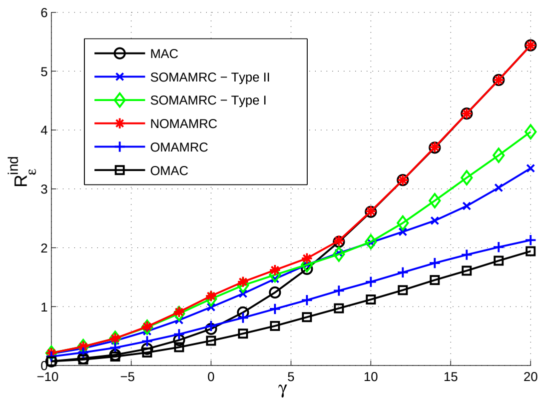

Fig. 5. $R_\epsilon^{ind}(\gamma)$ where $M = 4$, $L = 2$, $n = m_r = 1$, and $m_d = 4$.

antennas at the destination and to increase the reliability of source to relay links by 20 dB. It yields, for SOMAMRC Type I, $n = 1$, $m_r = 1$, and $m_d = 4$, while $\gamma_{r,d} = \gamma$, $\gamma_{s,r} = 100\gamma$, and $\gamma_{s,d} = \gamma$. As a result, for OMAMRC, we have $\gamma_{r,d} = \gamma$, $\gamma_{s,r} = 100M\gamma$, and $= \gamma_{s,d} = M\gamma$. For SOMAMRC Type II, we have $\gamma_{r,d} = \gamma$, $\gamma_{s,r} = 100M\gamma$, and $\gamma_{s,d} = \frac{\alpha}{\alpha/M+\bar{\alpha}}\gamma$. For NOMAMRC, we have $\gamma_{r,d} = \gamma$, $\gamma_{s,r} = 100\gamma$, and $\gamma_{s,d} = \alpha\gamma$. Fig. 5 shows the symmetric individual ϵ-outage achievable rate R_ϵ^{ind} for different schemes when the number of the relays in the system are $L = 2$. The same observation could be seen as in the previous set of simulation except that the SOMAMRC Type I has always better achievable rate than SOMAMRC Type II, which was not the case before, but there is a considerable range of SNR where both schemes have the same achievable rate.

V. Conclusions

In this paper, we have formulated the individual and common outage event for several relay assisted cooperative schemes for MAMRC conditional on the Selective Decode and Forward (SDF) relaying strategy. It reveals quite clear that removing (even partially) the constraint of orthogonality between links is an effective way of improving the spectral efficiency of such schemes. From a practical point of view, semi-orthogonal schemes can achieve an interesting trade-off between complexity and performance. For example, if the relay complexity has to be kept as low as possible, the SOMAMRC type II scheme is well adapted. Future work will be devoted to design practical transceivers with performance close to the theoretical limits presented in this paper.

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