Co-primary inter-operator spectrum sharing using repeated games

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Abstract—Currently, mobile network operators are allocated spectrum bands on an exclusive basis. While this approach facilitates interference control, it may also result in low spectrum utilization efficiency. Inter-operator spectrum sharing is a potential method to enhance spectrum utilization. In order to realize it, a protocol to coordinate the actions of operators is needed. In this paper, we propose a spectrum sharing protocol which is distributed in nature, does not require operator-specific information exchange, and incurs minimal communication overhead between the operators. Operators are free to decide whether they share spectrum or not as the protocol is based on a book keeping method is extended in [5] to incorporate component carrier (BIM) representing the inter-cell interference condition. The base stations exchange their background interference matrices with the neighboring cells. For interference control, neighboring operators help them to reduce the blocking probability and frame delay [1], [2]. Time domain spectrum sharing between different WiMAX systems is already a part of the IEEE 802.16h standard under the coordinated coexistence mode [3].

In [4] interference control is realized in frequency domain using the principle of carrier aggregation. Femtocells select autonomously and distributively primary component carriers (PCCs) to ensure coverage and secondary component carriers (SCCs) to enhance capacity provided that the operators are willing to exchange information and cooperate honestly. Under this requirement, many spectrum sharing algorithms are available in the literature. They differ related to the domain where inter-operator interference is handled i.e. time, frequency, and/or space [1]–[7].

Inter-operator spectrum sharing in time domain requires good time synchronicity between the operators. Operators with a low load could borrow their timeslots to heavily loaded operators helping them to reduce the blocking probability and frame delay [1], [2]. Time domain spectrum sharing between different WiMAX systems is already a part of the IEEE 802.16h standard under the coordinated coexistence mode [3].

In [4] interference control is realized in frequency domain using the principle of carrier aggregation. Femtocells select autonomously and distributively primary component carriers (PCCs) to ensure coverage and secondary component carriers (SCCs) to enhance capacity provided they do not disturb the neighboring cells. For interference control, neighboring base stations exchange their background interference matrices (BIM) representing the inter-cell interference condition. The method is extended in [5] to incorporate component carrier (CC) specific BIM. An upper bound on the sum-capacity of two operators is found in [6] assuming that operators exchange their user-specific channel quality indicators over all shared channels.

Inter-operator spectrum sharing in space has been considered in [7] where each operator is modeled as a transmitter-receiver link. It is assumed that operators exchange their channel state information and utilize cooperative transmit beamforming to steer their antenna beams towards the desired receiver.

Cooperative algorithms are useful to assess the maximum possible gains attained by inter-operator spectrum sharing. The setting in multi-operator spectrum sharing is fundamentally game theoretic, however, with the operators being a priori competing entities, and thus non-cooperative. As non-cooperative one-shot games may result in inefficient resource allocation for some if not for all players [8], one should look for other alternatives.

Operators are expected to share spectrum for a long time. Due to the fact that an operator has a persistent and publicly known identity, the operators can learn each other’s behavior. Accordingly, the interaction between operators would rather be modeled by repeated games [8]–[10]. A common assumption in [8]–[10] is that operators agree in advance about the spectrum allocation e.g. at a Nash equilibrium of the repeated game in [8], at a socially optimal point in [9], or at an orthogonal spectrum allocation in [10]. The operators have a sensing mechanism to detect deviation from the agreed allocation, and trigger a punishment mechanism when deviations are identified. In [8], [9] the punishment mechanism enforces the spectrum allocation of a one-shot game forever so that no player has incentive to deviate. This kind of strategy is strict because it does not incorporate forgiveness and punishes all players even if a single player deviates. In [10] a one-shot game is enforced for a finite time interval optimized to eliminate the incentive for deviation.

The approaches in the literature are based on the assumption that an operator knows the true state of the Radio Access Network (RAN) of other operators, so that an agreement about fair resource use can be identified, and a deviation from an agreed optimum point can be distinguished. Operators may, however, not want to share operator-specific information with competitors. An operator may have proprietary know-how in operating their RAN, and in differentiated service provisioning to their customers. In such a scenario, spectrum sharing can be based on monetizing the spectrum usage, and arranging auctions determining the usage of the spectrum.

In this paper, we consider spectrum sharing in a setting where no RAN information is revealed to other operators. We assume that operators are not willing to monetize spectrum use, keeping spectrum sharing on the RAN level. We illustrate that a repeated game can be setup so that both operators achieve better performance in comparison with static spectrum allocation, without revealing any operator-specific information nor making any agreements beforehand. We propose a coordination protocol that realizes spectrum sharing by means of book keeping of spectrum usage favors asked and received by the operators. In this perspective, operators are free to decide whether they take part into the game or not. Unlike the proposed spectrum
sharing rules in [8] and [9], the proposed protocol incorporates forgiveness resembling a tit-for-tat retaliation strategy [11]. Also, it does not fix the spectrum allocation as in [8]–[10] but it allows a flexible spectrum use based on inter-operator interference situation and traffic conditions. By using it in a scenario with two operators we are able to show that under load asymmetry both operators can benefit as compared to a scheme where no spectrum sharing is allowed.

II. SYSTEM MODEL

We consider a deployment scenario with two Mobile Network Operators (MNOs), Operator A and Operator B, sharing spectrum on a co-primary basis [12]. More precisely, the operators participate in a limited spectrum pool. The aggregate spectrum available for the operators is divided into $K$ CCs having equal bandwidths. Each operator is allocated one PCC to ensure coverage while the remaining SCCs form a shared spectrum pool. At a particular time instant, the distribution of users is modeled by a Poisson Point Process (PPP) with mean $N_A$ for Operator A and $N_B$ for Operator B.

The proposed coordination protocol can be used to negotiate the usage of the SCCs in the downlink. It is assumed that the base stations always transmit at maximum power level $P_t$ per CC no matter where their users are located. Power control and operator-specific radio resource management could also be incorporated into the system model but they will not change the basic functionality of the proposed coordination protocol. The noise power level per CC is $P_N$.

We assume that an operator can construct a network utility, that is a number that describes the quality of service (QoS) offered to its users. The network utility function can, for instance, be defined as a linear combination of average and cell edge performance. For simplicity, in this paper, we assume that the operators utilize a proportionally fair utility function directly constructed from the user rates. For $N_X$ users and $K_X$ CCs, the proportional fair network utility for Operator X is

$$U_X = \sum_{n=1}^{N_X} \log \left( \sum_{k=1}^{K_X} \frac{r_{n,x}^k}{g_{n,x}} \right)$$

(1)

where $r_{n,x}^k$ is the transmission rate of the $n$-th user of Operator X on the $k$-th CC. Note that the proposed coordination protocol does not require that the operators employ the same utility function nor that the operators are aware of each other’s utility function.

Let us consider that a CC is a resource that is shared in time among users belonging to the same operator. Different users belonging to the same operator cannot be allocated on the same resource simultaneously. If the inter-operator interference is treated as white noise and the $n$-th user of Operator X is scheduled on the $k$-th CC for a fraction $w_{n,x}^k$ of time, the rate of that user would be

$$r_{n,x}^k = w_{n,x}^k \cdot B \cdot \log_2 \left( 1 + \frac{g_{n,x}^k}{\gamma_{\text{eff}}} \right)$$

(2)

where $B$ is the bandwidth of a CC, $g_{n,x}^k$ is the downlink user SINR on the $k$-th CC and $\gamma_{\text{eff}}$ is the SINR efficiency loss due to system impairments.

Let us denote the useful signal link gain for the $n$-th user of Operator X on the $k$-th CC by $g_{n,x}^k$. Also, let us denote by $I_{n,x}$ the sum of the interference link gains scaled with the transmit power level $P_t$. Then, the downlink user SINR is

$$\gamma_{n,x}^k = \frac{P_t \cdot g_{n,x}^k}{P_N + I_{n,x}}$$

(3)

where the aggregate interference level should incorporate both own and other operator’s interfering base stations, subject to possible inter-cell interference control applied in the own network.

The scheduling weights, $w_{n,x}^k$, for Operator X are determined to maximize the utility $U_X$.

$$\text{Maximize : } U_X.$$  

$$\text{Subject to : } \sum_{n=1}^{N_X} w_{n,x}^k = 1 \forall k, w_{n,x}^k \geq 0, \forall \{n, k\}.$$  

(4)

In order to evaluate the effect the opponent operator has on the utility, an operator should measure the amount of interference it receives from the opponent. For downlink transmissions, this functionality can be a simple extension of LTE handover measurements. For example, the operator may ask its users to measure the interfering signal levels and report them to the serving base station. Note that this kind of functionality does not require any signaling between serving and interfering base stations. A similar approach has been adopted in [4] for the autonomous selection of PCC and SCC by femtocells belonging to the same operator.

III. COORDINATION PROTOCOL

In small cell deployments spectrum utilization can be enhanced if the allocation of SCCs varies according to the load and interference situation. For instance, an operator with low load can satisfy its QoS targets with few SCCs. The remaining SCCs can be used by another operator that has many users to serve. There should be an incentive for the low load operator to do this.

Without going to monetary transactions, we propose to regulate allocation of SCCs by means of spectrum usage favors asked and granted by the operators. Realizing spectrum sharing in the form of favors is a new concept that carries many important attributes. The operators do not have to reveal their performance metrics, QoS targets, etc. to other parties. In addition, they are not forced to take action. In a spectrum sharing scenario where there is a shared pool of SCCs we consider a single type of favor—an operator may ask the opponent for permission to start using an SCC on an exclusive basis.

A. One-shot game

To properly understand the game theoretical ramifications of the problem, we first consider a one-shot game where the players are two operators, and where no operator-specific information is revealed to the opponent. The game is strategic, and non-cooperative. The actions of each player are as follows: (i) to ask for a favor (ii) not to ask for a favor, but granting a favor if the opponent asks, or (iii) not to ask for a favor, but not granting a favor if the opponent asks. The outcomes of the game are such that a combination of one playing (i) while the other plays (ii) leads to a favor exchanged, while all other combinations lead to no favors exchanged.

For each player, the rewards depend only on its own current network realization: (i) the reward when a player takes a favor
is the utility gain when the interference on a SCC is eliminated (ii) the reward when a a player grants a favor is the utility loss when stopping to use a SCC and (iii) the reward when a player does not ask nor grant a favor is zero. We assume incomplete information so that the network realization of the opponent is not known, the only information that the players exchange is the indication of their actions.

For the formulated game it is easy to see that the only Nash equilibrium in the game is one where a player always asks but never grants a favor. As a result, in the one-shot game, both operators would utilize all SCCs irrespective of the load and interference conditions. This game is related to a public good game, which again is related to the well-known prisoner’s dilemma, see e.g. [13]. Note that cooperative game theory is not applicable in this setting. Due to the incomplete information of the other player’s network, the players would not know how to divide the benefits once a coalition is formed.

B. Repeated game

Since the operators are expected to share spectrum for a long time, and in many different network states, the one-shot game above would be played repeatedly. In a repeated game, in each stage game, the set of actions is same as in the one-shot game. This game is played multiple times. The outcomes of a stage game are realized for a fixed period of time, until the outcome of the next stage game takes force. In a repeated game, the particular action selected by a player may also depend on the other players’ previous actions [13]. The player may choose a strategy where an action is selected not only based on the instantaneous reward but also on the sequence of previous rewards. However, due to the incompleteness of the information, and due to the random process characterizing the network states, strategies based on assumption of cooperation, and punishing defections, do not apply. Cooperation happens in time domain, over multiple stages, by exchanging favors.

At each stage of the game, an operator may compute its utility gain if it gets a favor and its utility loss if it grants a favor. Let us denote by \( f_G \) and \( f_L \) the probability distribution functions (PDFs) of utility gains and utility losses respectively for an operator. Note that a priori these probability distributions do not depend on the willingness of the opponent to cooperate or not. The probability distribution is characterized by the point processes governing the location and needs of the own network users, and possibly some distributions governing the activity of the opponent network. In any network configuration there is a measurable gain due to reduced interference on a CC due to a getting a favor, and a loss due to granting a favor and not using one CC. The distribution then reflects the probability of the underlying network state. However, straightforward ways to estimate these distributions may depend on the willingness of the opponent to cooperate.

Here we consider a simplified strategy where the Operator X asks for a favor if its immediate utility gain is higher than a threshold \( \theta_0^{(X)} \). As a result, the probability that Operator X asks for a favor is

\[
Pr_{\text{ask}}^{(X)} = \int_{\theta_0^{(X)}}^{\infty} f_G^{(X)}(g) \, dg. \tag{5}
\]

Similarly, the operator grants a favor if its immediate utility loss is smaller than a threshold \( \theta_1^{(X)} \). Taking into account the fact that an operator cannot ask and grant a favor at the same stage of the game, the probability to grant a favor for operator X is

\[
Pr_{\text{grant}}^{(X)} = \int_0^{\theta_1^{(X)}} f_G^{(X)}(g) \, dg \cdot \int_0^{\theta_1^{(X)}} f_L^{(X)}(l) \, dl. \tag{6}
\]

where it is assumed that the distributions of utility gains and utility losses are independent.

In order to maintain reciprocity, we assume that the operators should grant about the same number of favors i.e.

\[
Pr_{\text{ask}}^{(A)} \cdot Pr_{\text{grant}}^{(B)} = Pr_{\text{ask}}^{(B)} \cdot Pr_{\text{grant}}^{(A)} \tag{7}
\]

where the left-hand side in Eq. (7) describes the probability that Operator A gets a favor and the right-hand side the probability that Operator B gets a favor.

An operator can monitor the probabilities of asking and granting of the opponent and set its own decision thresholds for satisfying the above constraint. However, there may be multiple combinations of thresholds fulfilling the constraint. We propose to identify the pair of thresholds maximizing an excess utility \( \tilde{U} \) calculated over the Nash equilibrium of the one-shot game. The excess utility for an operator reflects its expected utility losses are independent.

\[
\tilde{U}_A = Pr_{\text{grant}}^{(B)} \int_0^{\theta_1^{(A)}} f_G^{(A)}(g) \, dg - Pr_{\text{ask}}^{(B)} \int_0^{\theta_1^{(A)}} f_L^{(A)}(l) \, dl. \tag{8}
\]

In order to solve the optimization problem (8) for Operator A, we construct the Lagrangian function

\[
\mathcal{L}_A = \tilde{U}_A - \lambda_A \cdot \left( Pr_{\text{ask}}^{(A)} \cdot Pr_{\text{grant}}^{(B)} - Pr_{\text{ask}}^{(B)} \cdot Pr_{\text{grant}}^{(A)} \right) \tag{9}
\]

where \( \lambda_A \) is the Lagrangian multiplier.

Possible solutions of the optimization problem (8) can be identified by solving the system of first-order conditions. Taking the partial derivative of the Lagrangian in terms of \( \theta_0^{(A)} \) and setting it equal to zero allows us to compute the value of the Lagrangian multiplier \( \lambda_A = \theta_0^{(A)} \). Setting the partial derivative of the Lagrangian with respect to \( \theta_0^{(A)} \) equal to zero and substituting the value of the Lagrangian multiplier into the resulting equation gives

\[
Pr_{\text{grant}}^{(A)} \left( \theta_0^{(A)} - \theta_1^{(A)} \right) = Pr_{\text{ask}}^{(B)} \int_0^{\theta_1^{(A)}} f_L^{(A)}(l) \, dl. \tag{10}
\]

Note that from this one can deduce that \( \theta_0^{(A)} > \theta_1^{(A)} \). The pair of decision thresholds that may maximize the Lagrangian must jointly satisfy (11) and the constraint in (7) which can be rewritten as

\[
Pr_{\text{grant}}^{(B)} \int_0^{\theta_1^{(A)}} f_G^{(A)}(g) \, dg = Pr_{\text{ask}}^{(B)} \int_0^{\theta_1^{(A)}} f_G^{(A)}(g) \, dg \int_0^{\theta_1^{(A)}} f_L^{(A)}(l) \, dl. \tag{12}
\]
Fig. 1. Indoor inter-operator deployment scenario. Different colors represent base stations of different operators.

Even though the system of equations (11,12) does not allow a closed-form solution, it is straightforward to solve numerically. Let us denote the pair of thresholds solving this system by \((\theta^*_g(A), \theta^*_l(A))\). Also, \(\lambda^*_A = \theta^*_g(A)\). It is possible to show that the bordered Hessian of the Lagrangian at the stationary point \((\theta^*_g(A), \theta^*_l(A), \lambda^*_A)\) is positive and thus this is a local maximizer. In the Appendix, we show that the interior solution, \((\theta^*_g(A), \theta^*_l(A))\), satisfies \(U_A > 0\). Therefore, using decision thresholds calculated based on the system of Eq. (11) and (12) results in better performance in comparison with the Nash equilibrium of the one-shot game. Besides the calculation of the Lagrangian at the stationary point, we also compute it at the borders. The only border condition that results in non-trivial solution sets the decision threshold on the distribution of losses equal to infinity, \(\theta^*_l(A) \to \infty\), and optimizes the threshold on the distribution of gains \(\theta^*_g(A)\). Finally, the point, either interior or border, that maximizes the Lagrangian is selected.

IV. NUMERICAL EXAMPLES

In order to assess the performance of the proposed coordination protocol, we consider an indoor deployment scenario in a hall of a single story building. The hall is a square with a side of 50 m. The base stations are partitioned into two groups as illustrated in Fig. 1 modeling a spectrum sharing scenario with two operators. The service areas of the operators fully overlap. A user is attached to the base station of its home network giving the highest signal level at its location.

We consider a power law model for distance-based pathloss with attenuation constant \(10^{-4}\) and pathloss exponent 3.7. Fading is not modeled. The available power budget on a CC is 20 dBm, the thermal noise power is \(-174\) dBm/Hz and the noise figure is 10 dB. The SINR efficiency in equation (2) is \(\gamma_{eff} = 2\). There are in total three CCs each with bandwidth \(B = 20\) MHz. The shared spectrum pool consists of one SCC.

First, we consider an initialization phase of 200 000 simulation time units (or equivalently 200 000 stage games). At each stage game, the operators calculate and keep track of their utility gain and loss in order to progressively construct the distributions of utility gains and losses. During the initialization phase, we simulate many different load combinations for the two operators so that the distributions of gains and losses at the end of the initialization can be seen as the steady state distributions, characterized by the full probability density of network states. In Fig. 2 we depict the distributions of gains and losses for Operator A at the end of the initialization.

Next, we evaluate the performance of the proposed scheme in terms of the user rate distribution over a finite time horizon of 20 000 time units following the initialization phase. In the beginning, the values of the thresholds are set arbitrarily \(\theta_g = \theta_l = 2\) for both operators. Every 100 stage games the thresholds are updated by solving the optimization problem in Eq. (8).

First, we consider a case with equal mean network loads for the two operators. The mean number of users for the operators are equal to \(N_A = N_B = 5\) over the full course of the simulation, 20 000 stage games. In Fig. 3 we see that Operator A improves its mean user rate by approximately 10 % using the proposed spectrum sharing scheme.

Next, we consider a scenario with load asymmetry between the operators. The mean number of users for the first 10 000 stage games are equal to \(N_A = 8\) and \(N_B = 2\). In the second half of the simulation, for the next 10 000 stage games, the mean values are reversed. Given the allocation of CCs at each stage of the game, the operator computes and keeps track of the user rates. Recall that granted favors are valid only for a particular stage game. At the end of the stage game, the CC allocation falls back to the original state i.e. both operators utilize the SCC. The performance of the proposed scheme is assessed in comparison with a static spectrum allocation scheme where both operators fully utilize the SCC, i.e. to the Nash equilibrium of the one-shot game. In Fig. 4, the rate distribution for the users of Operator A is depicted over the full course of the simulation. In the first 10 000 stage games, Operator B can mostly cope without the SCC due to the low number of users and it grants more favors than the Operator A. In the second half of the simulation, Operator A returns the favors. Overall, Operator A offers better QoS in comparison with the QoS attained using static spectrum allocation, e.g., it improves its mean user rate by approximately 21 %. The user rate distribution curves for Operator B follow the same trend.

V. CONCLUSIONS

Existing coordination protocols for inter-operator spectrum sharing assume either full cooperation based on operator-
specific information exchange or that operators agree beforehand on some spectrum allocation which is maintained under the threat of punishment. These attributes can be problematic because on the one hand, MNOs are competing entities and on the other, static spectrum allocation performs poorly under network load variations. According to the coordination protocol proposed in this paper, MNOs with low load give spectrum usage favors to heavily-loaded operators. MNOs that have granted favors in the past are likely to return these favors in future and reciprocity is maintained. MNOs offer better QoS in comparison with static spectrum allocation without revealing any operator-specific performance indicators. In this paper we illustrated that the coordination protocol can be used to enhance the user rate performance in a spectrum sharing scenario with two operators over a limited spectrum pool. The coordination protocol takes advantage of the high inter-operator interference and load asymmetry between the operators. It is not clear what the Nash equilibrium of the repeated game in question would be, but our results show that a rational operator, knowing that the opponent is rational and has a network with similar characteristics, has incentive to be cooperative.

VI. APPENDIX

Based on Eq. (11), the expected loss for Operator A can be expressed as

\[
\theta_i^A \left( \frac{1}{f_L^A(l)} \right) dl = \theta_i^A \int f_L^A(l) dl \cdot P_{grant}^{(B)} \left( \theta_i^A \right) - \theta_i^A \left( \frac{1}{f_L^A(l)} \right) dl \cdot P_{ask}^{(B)} \left( \theta_i^A \right). \tag{13}
\]

Based on Eq. (12), the probability to ask for a favor can be read as

\[
\int_0^{\theta_i^A} f_G(g) dg = \frac{P_{grant}^{(B)} \int_0^{\theta_i^A} f_G(g) dg}{P_{grant}^{(B)} \int_0^{\theta_i^A} f_G(g) dg + P_{ask}^{(B)} \int_0^{\theta_i^A} f_G(g) dg}. \tag{14}
\]

After replacing Eq. (13) and (14) into Eq. (9), the excess utility can be written as

\[
\widetilde{U}_A = \frac{\left( P_{grant}^{(B)} \int_0^{\theta_i^A} f_G(g) dg \right)^2}{P_{ask}^{(B)}} \left( \theta_i^A - \theta_i^A \left( \frac{1}{f_L^A(l)} \right) dl \cdot P_{grant}^{(B)} \left( \theta_i^A \right) \right) f_L^A(l). \tag{15}
\]

Fig. 3. Rate distribution for the users of Operator A obtained by the proposed scheme using spectrum usage favors and also by a static spectrum allocation scheme where the operators always utilize the SCC.

Fig. 4. Rate distribution for the users of Operator A obtained by the proposed scheme using spectrum usage favors and also by a static spectrum allocation scheme where the operators always utilize the SCC.

which is always positive since \( \theta_i^A > \theta_i^A \).

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