Frequency Allocation in Non-Coherent Joint Transmission CoMP Networks

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Abstract*—In this paper, we study the problem of joint transmission (JT) in coordinated multipoint (CoMP) networks from a new point of view where the system performance is optimized via frequency allocation for 5G small cells. Moreover, we investigate the implementation of hybrid automatic repeat request (HARQ), as an efficient scheme facing the feedback load problem in CoMP setups. The results are obtained for the cases with slow and fast fading conditions. Considering the channel state information (CSI) only at the receiver, we show that at low and medium signal to noise ratios (SNRs) sharing the frequency resources between users outperforms the case when the frequency resources are dedicated under non-coherent JT-CoMP setting. We find that the maximum long term throughput is achieved by either sharing the entire frequency resources between the users or allocating each user in a disjoint dedicated frequency resource. These extreme cases show the best performance in the SNR region of interest. Finally, as demonstrated analytically and numerically, HARQ feedback increases the long term throughput and reduces the outage probability substantially, with an affordable average delay.

***Index Terms*—Dynamic Frequency Allocation, HARQ, JT-CoMP, Limited Feedback Systems, Open Loop Systems**

I. INTRODUCTION

The impetus of mobile data consumption has driven the need for amazingly fast ubiquitous communication systems. This is one of the main characteristics of 5th generation (5G) mobile and wireless systems. In [1], several scenarios with challenging requirements and performance indicators are derived for 5G communication systems. Typical user data rates are expected to be in the order of several Gbps in a dense urban environment. To achieve these rates, ultra dense wireless networks and hotspots need to be created, giving rise to small cells.

Coordinated multipoint (CoMP) transmission is a promising technique to improve the users' data rate [2]-[4]. CoMP is one of the technologies aiding to meet the challenging requirements of 5G communications. One variant is joint transmission (JT) CoMP where more than one base station (BS) is involved in jointly serving a user. JT-CoMP promises significant improvement in the data rate. However, in an ultra dense network, it poses tremendous problems. Firstly, providing all the coordinated BSs with channel state information (CSI) is difficult given a dense network deployment. Secondly, feeding back the CSI from the users to its serving BSs consumes the precious air interface resources wherein the control signals could easily overwhelm the network. In an ultra dense network having a network wide cooperation makes feedback extremely costly. Hence, limited or no feedback schemes are very attractive, as long as the system performance is not significantly deteriorated.

Hybrid automatic repeat request (HARQ) is an efficient method to boost throughput with limited or no CSI at the transmitter (CSIT). It is a simple method to inform the transmitter with acknowledgment/negative acknowledgment (ACK/NACK) of successful or unsuccessful decoding of the message at the receiver. HARQ is typically not considered in the theoretical analysis of CoMP setups, as it is generally thought to have large latency. However, HARQ is already present in many of the existing standards such as Long Term Evolution [5], WiMAX [6], and it will be a part of 5G systems [1]. Therefore, the latency involved with HARQ is unavoidable in many applications. Here, we show that, with limited feedback resources, implementing HARQ in the CoMP setups improves the system performance considerably with limited latency.

In an open loop system, the CSI is only available at the receiver (CSIR), thereby reducing the control signaling in the uplink. In [7], [8], Gaussian broadcast channel with CSIR is investigated for the single user case and the multiple input multiple output (MIMO) case, respectively. In [9], frequency diversity is collected by serving users on equidistant dedicated frequency resources. In relation to our work, dynamic spectrum management is studied in [10], where multiple users share the common frequency band and dynamically choose their transmit power spectral density based on the channel conditions. However, this requires CSIT. A closed loop system involving CSI feedback of channels to many neighboring nodes creates a substantial overhead in an ultra dense network deployment. To limit the feedback overhead, coarse CSI quantization might be needed, which may lead to errors in these estimates resulting in performance even worse than open loop systems [11].

In small cells, the delay spread is much smaller compared to the one experienced in macrocells. This results in a large coherence bandwidth where a large group of subcarriers experience frequency flat fading. With these large groups of frequencies now available, one can optimize how these can be allocated to the users. In our contribution, we look at the

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problem of JT-CoMP from a new point of view where the system performance is optimized via frequency allocation. We divide a given frequency bandwidth W into two parts, namely, *dedicated* and *shared* frequency resources. In the dedicated frequency resource, each user is served on disjoint frequency resources, while in the shared frequency resource, a subset of users are served on the same frequency resources. Then, we investigate the optimal partitioning between these two types of frequency resources in terms of system throughput or the users' outage probability, which characterizes the system performance.

The main contributions of the work are: 1) We investigate schemes for frequency allocation in JT-CoMP with CSIR only. 2) We show that depending on the transmission rate and power the maximum long term throughput, or the minimum outage probability, is obtained by either fully sharing the frequency resources (at low/medium SNRs) or dedicating the frequency resources disjointly to the users (at high SNR). We discover that there exists a switching point where the frequency resources can either be completely shared or completely dedicated for the SNR values of practical interest. 3) We incorporate HARQ as a fallback mechanism, and show that HARQ is a very efficient scheme for limited feedback CoMP setups, resulting in high long term throughput and low outage probability with an affordable average delay. The results are obtained for the fast and slow fading conditions where the channel changes in each (re)transmission and remains constant within the HARQ (re)transmissions, respectively. Particularly, we show that, with HARQ, a better system performance in terms of outage probability is observed in the case of fast fading, where the channel changes with each retransmission round, compared to slow fading, where the channel remains constant during retransmissions.

Some of the good points of the proposed scheme are: 1) adaptive power allocation is not required, which substantially simplifies the design of the power allocation problem. 2) It leads to considerably low feedback load, and 3) in contrast to the quantized CSI schemes, it is not sensitive to the unreliability in the feedback channel [12]. To summarize, the proposed scheme achieves some advantages related to coordinated data transmission but can still avoid many problems that may limit the practical implementation of CoMP networks. Therefore, the proposed scheme can be of interest in 5G communication setup, especially in dense urban information society [1]. Our results are of particular interest when we remember that, as shown in [13], optimal frequency allocation between the BSs/users is essential for cooperative networks operating at moderate/high signal to noise ratios (SNRs), to mitigate the out-of-cluster or the channel estimation error due to large cluster size/cooperating nodes.

The paper is organized as follows: the system model is proposed in section II, where a generic model is presented for the sharing and dedicating the frequency resources. Next the theoretical analysis on the long term throughput, outage probability, and average delay due to retransmission are given. The simulation results are presented in section III. The novel results are summarized and the future work concludes the paper in section IV. The notation used in this paper is summarized as follows: the expectation operator is denoted as $\mathbf{E}(\cdot)$. Sets are indicated in calligraphic letters such as $\mathcal{X}$.

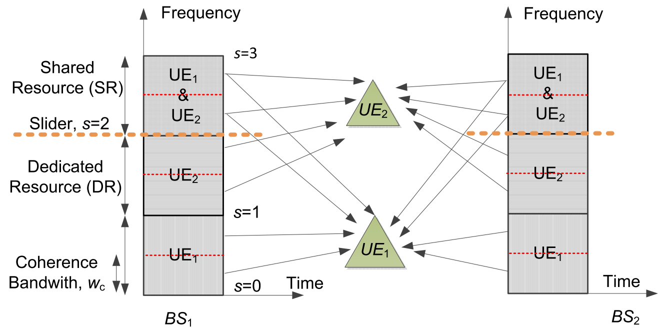

Fig. 1. An example of the system model with two users. BS k is serving $M = 2$ users, where the dedicated frequency resource for every mth user and the shared frequency resource between all the M users is illustrated. The slider s shows the way in which the frequency resources are allocated. In this figure, $s = 2$ and $N = 1$.

II. SYSTEM MODEL

In this work, we consider non-coherent JT-CoMP with CSIR, as the BSs only have the knowledge of the users to schedule and they have no knowledge of the channel, therefore, coherent JT is not a viable option. Then, we transition our focus on a closed loop system where only the ACK or NACK based on HARQ is available at the BS without any CSI feedback. Note that, in contrast to quantized CSI schemes, the HARQ protocols are very robust to unreliable feedback channels [12].

Consider K single antenna BSs serving M single antenna users from a set $\mathcal{M} = \{1, \ldots, M\}$ such that M is the cardinality of the set $\mathcal{M}$. When one or more groups of subcarriers are allocated to a particular user, these form a dedicated resource for that user. While sharing one or more groups of subcarriers for more than one user forms a shared resource. For example, the system model with two users is illustrated in Fig. 1.

To maintain fairness, each user gets equal amount of time-frequency resources. This is captured in the system model as shown in Fig. 1 using a *slider*. To explain how the slider functions, some preliminaries are introduced. The number of subcarriers that experience frequency flat fading is N, and they occupy a coherence bandwidth of w_c. To have a fair comparison for allocating frequency resources between users, we assume $w_c = \frac{W}{M(M+1)}$ where W is the total bandwidth. We consider a slider, s, that divides the frequency resources into dedicated and shared resource as shown in the Fig. 1. The possible positions of the slider in terms of the number of subcarriers is MNs where $s = 0, 1, 2, \ldots, (M + 1)$. The question here is, given a bandwidth W where should the slider be placed, to provide the highest achievable throughput with CSIR alone.

Let $m \in \mathcal{M}$ and the mth user data comprises of one long codeword that is split into two main parts $x^{\mathrm{DR}}_{m;n}$ and $x^{\mathrm{SR}}_{m;n'}$, where the $x^{\mathrm{DR}}_{m;n}$ is transmitted on the dedicated resource on the nth subcarrier while $x^{\mathrm{SR}}_{m;n'}$ is transmitted on the shared resource on the n'th subcarrier, such that $n \neq n'$ for a given slider value s. Note that the coded user data is unique in the allocated subcarrier and transmitted using Gaussian codebooks. In the shared resource, $x^{\mathrm{SR}}_{m;n'}$ and $x^{\mathrm{SR}}_{m^{'};n'}$ are combined in the analog domain where $m^{'} \neq m$. At the receiver, the user data is accumulated after combining the decoded data from all the allocated subcarriers for that user. In the following, we shall use the acronyms DR and SR for dedicated resource and shared resource, respectively. The signal received by the mth user for a given slider value of s is

$$y^{\mathrm{DR}}_{m;n} = \sum_{k=1}^{K} h_{m,k;n} \sqrt{P^{\mathrm{DR}}_{m;n}} x^{\mathrm{DR}}_{m;n} + n_0 \tag{1}$$

$$\begin{aligned} y^{\mathrm{SR}}_{m;n'} &= \sum_{k=1}^{K} h_{m,k;n'} \sqrt{P^{\mathrm{SR}}_{m;n'}} x^{\mathrm{SR}}_{m;n'} + \\ & \sum_{\substack{l \neq m \\ l \in \mathcal{L}_i}} \sum_{k=1}^{K} h_{m,k;n'} \sqrt{P^{\mathrm{SR}}_{l;n'}} x^{\mathrm{SR}}_{l;n'} + n_0, \end{aligned} \tag{2}$$

where the noise signals are complex Gaussian $n_0 \sim \mathcal{CN}\left(0, \sigma^2\right)$ with variance σ^2. The channel from the kth BS to the mth user in the nth subcarrier due to the transmission to mth user is $h_{m,k;n}$. The power allocated to the mth user is correspondingly $P^{\mathrm{DR}}_{m;n}$ and $P^{\mathrm{SR}}_{m;n'}$. The interference power due to the lth user is $P^{\mathrm{SR}}_{l;n'}$. Note that this model is generic as it provides room for sharing the frequency resources for the M users in various ways. Allocating these M users in the shared frequency resources becomes a combinatorial problem. To illustrate this point, let us define $\mathcal{C} = \{$A set of all combinations of users, such that every user is sharing with at least one other user$\}$. For example, $\mathcal{M} = \{1, 2\}$ and $M = 2$ leads to $\mathcal{C} = \{\{1, 2\}\}$. With $\mathcal{M} = \{1, 2, 3\}$ and $M = 3$, we have $\mathcal{C} = \{\{1, 2, 3\}, \{\{1, 2\}, \{2, 3\}\}, \{\{1, 3\}, \{2, 3\}\}, \{\{1, 2\}, \{1, 3\}\}\}$, where $\mathcal{L}_i \subseteq \mathcal{C}$ and the index i is used to identify a subset. With $i = 1$, we have $\mathcal{L}_1 = \mathcal{M}$ is the subset where all the users share the same frequency resource. The SNR for the mth user in the nth dedicated subcarrier is

$$\gamma^{\mathrm{DR}}_{m;n} = \frac{\frac{N}{w_c} \left| \sum\limits_{k=1}^{K} h_{m,k;n} \sqrt{P^{\mathrm{DR}}_{m;n}} \right|^2}{\sigma^2} \tag{3}$$

and the signal to interference plus noise ratio (SINR) in the shared resource is

$$\gamma^{\mathrm{SR}}_{m;n'} = \frac{\frac{N}{w_c} \left| \sum\limits_{k=1}^{K} h_{m,k;n'} \sqrt{P^{\mathrm{SR}}_{m;n'}} \right|^2}{\frac{N}{w_c} \left| \sum\limits_{\substack{l \neq m \\ l \in \mathcal{L}_i}} \sum\limits_{k=1}^{K} h_{m,k;n'} \sqrt{P^{\mathrm{SR}}_{l;n'}} \right|^2 + \sigma^2}. \tag{4}$$

where $P^{\mathrm{SR}}_{m;n'}$ and $P^{\mathrm{SR}}_{l;n'}$ are chosen such that the power is uniformly distributed between the users. In general, based on [14, eq. 9.62] the rate achieved by the user with only the dedicated resource can be written as:

$$R^{\mathrm{DR}}_{m} = \sum_{\substack{n=(m-1)sN+1 \\ s \neq 0}}^{msN} \frac{w_c}{N} \log_2\left(1 + \gamma^{\mathrm{DR}}_{m;n}\right). \tag{5}$$

Note that in (5), the transmit power in each of the nth subcarriers is $P^{\mathrm{DR}}_{m;n} = \frac{P}{M(M+1)N}$ under uniform power allocation (not necessarily optimal), where P is the total power in the system, and each BS uses P/K Watts of power.

With the shared resource, the M users are served on the same frequency resource, such that the useful signal of the mth user is affected by interference from the other users. This interference is treated as noise. The rate achieved by the mth user with only shared resource is

$$R^{\mathrm{SR}}_{m} = \sum_{\substack{n'=MsN+1 \\ s \neq M+1}}^{M(M+1)N} \frac{w_c}{N} \log_2\left(1 + \gamma^{\mathrm{SR}}_{m;n'}\right), \tag{6}$$

Therefore, the maximum achievable rate for the mth user is $R_m = R^{\mathrm{DR}}_{m} + R^{\mathrm{SR}}_{m}$. Note that when $s = 0$, $R^{\mathrm{DR}}_{m} = 0$ as $y^{\mathrm{DR}}_{m;n}$ does not exist, and when $s = M + 1$, $R^{\mathrm{SR}}_{m} = 0$ as $y^{\mathrm{SR}}_{m;n'}$ does not exist.

To calculate the long term throughput, we assume HARQ with total number of (re)transmissions as $T + 1$, where first transmission could be followed by a maximum of T retransmissions. We focus on the continuous data communication model where the BS have a large pool of information that needs to be continuously sent to the users [15], [16]. Using the renewal-reward theorem [17], the long term throughput is

$$\begin{aligned} \eta &= \frac{\text{Total number of successfully decoded bits}}{\text{Total number of channel uses (time)}} & (7) \\ &= \sum_{m=1}^{M} \mathbf{E}(R_m), & (8) \end{aligned}$$

where the last equality comes from the continuous communication assumption [15, Section IV.B] and $\mathbf{E}(R_m)$ is the expected achievable rate of the mth user.

As the best HARQ protocol, reaching the highest throughput [15], [17], we concentrate on the incremental redundancy (INR) HARQ protocol. With INR, the data is encoded into a *parent* codeword which is then punctured into $T + 1$ sub-codewords. In each retransmission round, a new sub-codeword is sent from the BSs if requested. In other words, the data is first sent with rate r_m and it is decoded if $R_m > r_m$. Otherwise, a new sub-codeword is retransmitted and the data rate reduces to $\frac{r_m}{2}$. In this way, the data is decoded in the second round if $\frac{r_m}{2} < R_m \leq r_m$. Following the same procedure, the long term throughput of the M users for $T+1$ (re)transmissions is

$$\eta_{T+1} = \sum_{m=1}^{M} \sum_{t=1}^{T+1} \frac{r_m}{t} \Pr\left(\frac{r_m}{t} < R_m \leq \frac{r_m}{t-1}\right). \tag{9}$$

Setting $T = 0$, the results are simplified to the case with no HARQ feedback, i.e, open loop communication. Note that the long term throughput increases with the number of retransmissions T.

The outage probability after the $(T+1)$th retransmission for the mth user is

$$P_{m,T+1}^{\text{outage}} = \text{Pr}\left(R_m \leq \frac{r_m}{T+1}\right). \tag{10}$$

The average delay with a maximum of $T+1$ (re)transmissions for the mth user is found to be

$$\begin{aligned} \overline{D}_{m,T+1} &= \sum_{t=1}^{T} tp\text{Pr}\left(\frac{r_m}{t} < R_m \leq \frac{r_m}{t-1}\right) \\ &\quad + p(T+1)\text{Pr}\left(R_m \leq \frac{r_m}{T}\right), \end{aligned} \tag{11}$$

where p seconds is the round trip delay, i.e., the length of the sub-codewords sent in the HARQ (re)transmissions.

Finally, note that the maximum achievable rate of the mth user, i.e., R_m in (9)-(11) comes from (3)-(6) in which we have assumed that the channel remains constant during the (re)transmissions. We refer to this model as slow fading, modeling slow moving users [18]. However, as shown in [18], it is straightforward to extend the results to the fast fading model, for high speed users, where the channel changes in each round. In that case, the channel terms of (3)-(6) will change independently in each (re)transmission.

III. SIMULATION RESULTS

For an illustrative example, we focus on the $M = 2$ users case because it provides good insight for the general case with arbitrary number of users. With $M = K = 2$ and the number of subcarriers experiencing a flat fading Rayleigh channel is $N = 1$, the received signal for the mth user in the shared resource becomes

$$\begin{aligned} y_{m;n'}^{\text{SR}} = &\sqrt{\frac{P}{2(2+1)}\frac{1}{2}}\sum_{k=1}^{2} h_{m,k;n'} x_{m;n'}^{\text{SR}} + \\ &\sqrt{\frac{P}{2(2+1)}\frac{1}{2}} \sum_{\substack{l=1 \\ l\in\mathcal{M}, l\neq m}}^{2} \sum_{k=1}^{2} h_{m,k;n'} x_{l;n'}^{\text{SR}} + n_0, \end{aligned} \tag{12}$$

where P corresponds to the SNR, assuming the noise variance $\sigma^2 = 1$. The rate achieved by the mth user upon sharing the frequency resource is

$$R_m^{\text{SR}} = \sum_{\substack{n'=2s+1 \\ s\neq M+1}}^{2(2+1)} w_c \log_2\left(1 + \frac{A}{B+\sigma^2}\right), \tag{13}$$

where

$$\begin{aligned} A &= \frac{P}{2(2+1)}\frac{1}{2}\frac{1}{w_c}\left|\sum_{k=1}^{2} h_{m,k;n'}\right|^2 \\ B &= \frac{P}{2(2+1)}\frac{1}{2}\frac{1}{w_c}\left|\sum_{\substack{l=1 \\ l\in\mathcal{M}, l\neq m}}^{2} \sum_{k=1}^{2} h_{m,k;n'}\right|^2. \end{aligned}$$

With shared resource between M users, the transmit power is scaled by a factor of $\frac{1}{M}$ in every shared resource. Finally, the rate achieved by dedicating disjoint frequency resource to the mth user becomes

$$R_m^{\text{DR}} = \sum_{\substack{n=(m-1)s+1 \\ s\neq 0}}^{ms} w_c \log_2\left(1 + \frac{1}{\sigma^2 w_c}\frac{P}{2(2+1)}\left|\sum_{k=1}^{2} h_{m,k;n}\right|^2\right), \tag{14}$$

which, along with (9)-(11) and (13) are used to determine the system performance in different conditions. For simulations, we assume that the channel experienced by the users from various frequency resources follows an independent and identically distributed Rayleigh fading distribution. The total bandwidth is normalized to $W = 1$ Hz and the possible ways to allocate the frequency for the mth user is as shown in Table I, where the slider at $s = 0$ allows complete sharing of the frequency resources. At the other extreme, $s = 3$ allows dedicated and disjoint frequency resources to the users. With $s = 1$, fewer frequency resources are dedicated than being shared. With $s = 2$, the amount of dedicated and shared resources are the same for the mth user (please see Fig. 1 for possible placements of the slider s). The table also captures the legends in the forthcoming figures and their interpretation.

Table I

AN ILLUSTRATION OF THE POSSIBLE FREQUENCY ALLOCATIONS FOR THE mTH USER WHEN $M = K = 2$. DR AND SR DEFINE THE POSSIBLE RATIOS IN WHICH THE FREQUENCY RESOURCES, $W = 1$ HZ, ARE DEDICATED AND SHARED FOR THE mTH USER.

Slider pos.	DR	SR	Interpretation	Legend
$s = 0$	0	1	All shared	Blue asterisk
$s = 1$	1/6	4/6	Some dedicated	Magenta crosses
$s = 2$	2/6	2/6	Uniformly allocated	Red dots
$s = 3$	3/6	0	Dedicated	Black triangles

The results are presented for different SNRs $10\log_{10}\left(P/\sigma^2\right)$ with $\sigma^2 = 1$. For an SNR $= 0$ dB and $T = 0$, Fig. 2 shows the long term throughput of the open loop setup versus the transmission rate, r_m. Note that, we do not bias towards any user, hence we consider $r_m = r_{m'}, \forall m \neq m'$. It can be observed that, with SNR = 0 dB, the shared resource produces the best long term throughput without HARQ.

Optimizing the transmission rate, the long term throughput achieved with dedicated resource increases with SNR as shown in the Fig. 3. However, the shared resource flattens out at high SNR. This can be explained using (13) where at high SNR, the noise becomes negligible compared to interference, $B \gg \sigma^2$. The useful term A and the interference term B are proportional to P. Therefore, R_m^{SR} becomes independent of P and the blue asterisk curve for shared resource saturates at high SNR. Thus, the diversity gain [18, eq. (14)] is determined by the dedicated resource part at asymptotically high SNRs if uniform power allocation is implemented by the BSs. It is also interesting to note that the highest long term throughput is achieved only in the two extreme cases, i.e., when the

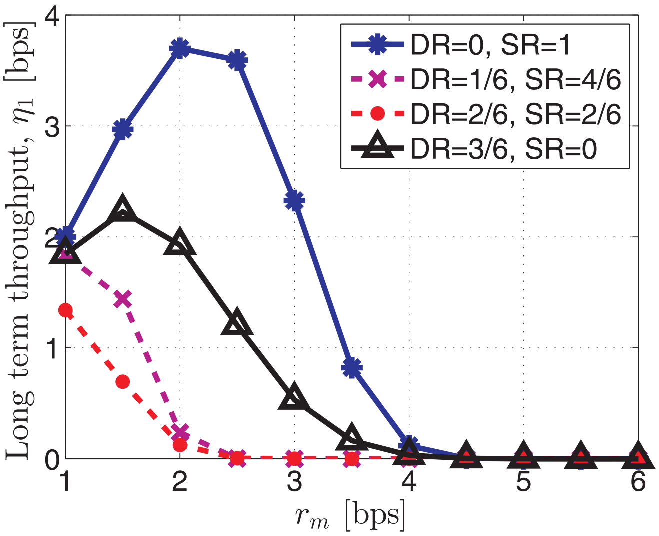

Fig. 2. Long term throughput η_1 versus r_m for SNR $= 0$ dB and $T = 0$.

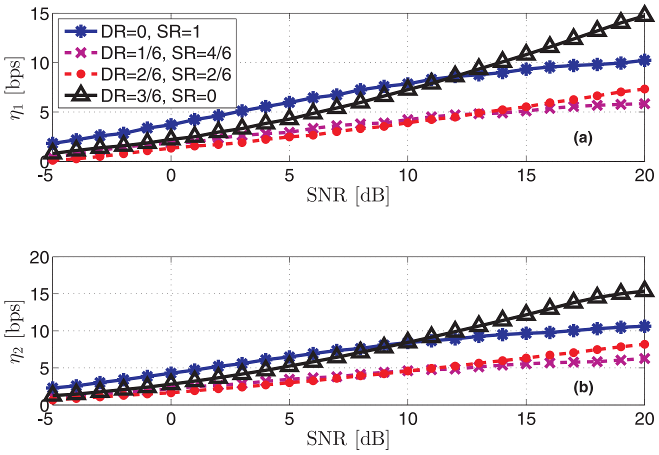

Fig. 3. Plot of long term throughput, η_1 in subplot (a) and η_2 in subplot (b). For each SNR and frequency allocation scheme, the initial transmission rate r_m is optimized to maximize the long term throughput under slow fading conditions.

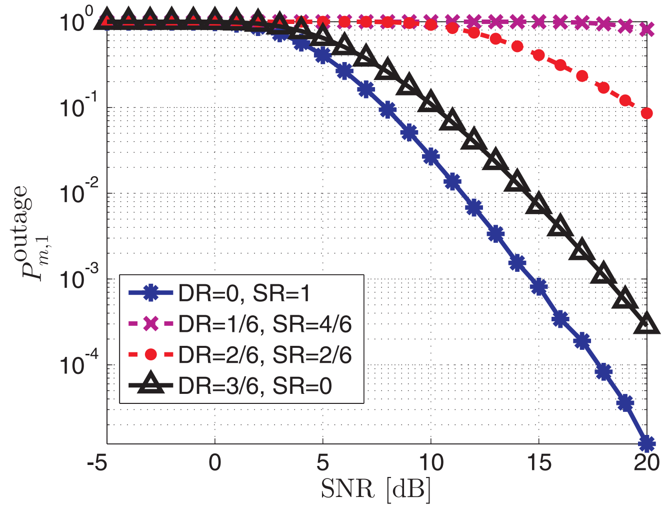

Fig. 4. Outage probability, $P_{m,1}^{\text{outage}}$, versus SNR, for given $r_m = 4$ bps in an open loop system (without HARQ, $T = 0$).

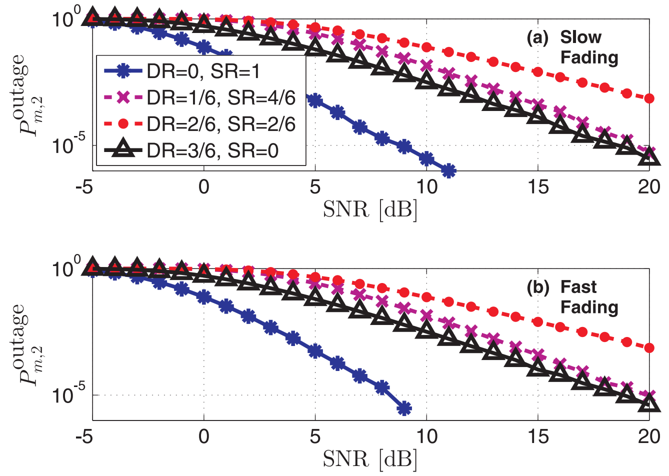

Fig. 5. Outage probability versus SNR, for given $r_m = 4$ bps in a closed loop system. The subplot (a) is the outage probability for the slow fading model where the channel remains constant during all (re)transmissions. The results for fast-fading, where the channel changes independently in each (re)transmission, are given in subplot (b).

frequency resources are either completely shared (at low and medium SNR) or completely dedicated (at high SNR) as shown in Fig. 3. That is, the optimal transmission scheme, in terms of long term throughput, is either to completely share or completely dedicate the frequency resources, and the optimal scheme is determined based on the SNR. In other words, there is a switching point between these two schemes in the SNR range of interest. Note that the results presented here also hold for $M > 2$ as long as the number of users sharing a given frequency resource is two, depending on $\mathcal{L}_i$. With HARQ INR, the above analysis also holds for different number of retransmissions T as well. Also, the long term throughput increases with T, due to accumulative nature of (9).

Fig. 4 shows the outage probability, $P_{m,1}^{\text{outage}}$, for the case without HARQ, i.e., an open loop system. While Fig. 5 shows the outage probability, $P_{m,2}^{\text{outage}}$, in a closed loop system with one bit HARQ feedback for the slow fading model. It can be observed that compared to the open loop system, more diversity can be extracted when HARQ is applied. It is more prominent in the fast fading case and agrees well with the literature [18]. Note that with fast fading conditions, the INR can reach the ergodic capacity when $T \to \infty$ [17].

Any sort of feedback gives rise to delay in the system. We study the average delay incurred due to the HARQ mechanism for a chosen transmission rate, r_m. In particular, we focus on SNR $= 0$ dB and 10 dB with $p = 1$ (normalized) in (11). The average delay is evaluated for various values of r_m. In Fig. 6, when the initial transmission rate is low, with low SNR of 0 dB, the blue asterisk curve showing the shared frequency resources performs better than the black triangles curve where the frequency resource is dedicated in terms of

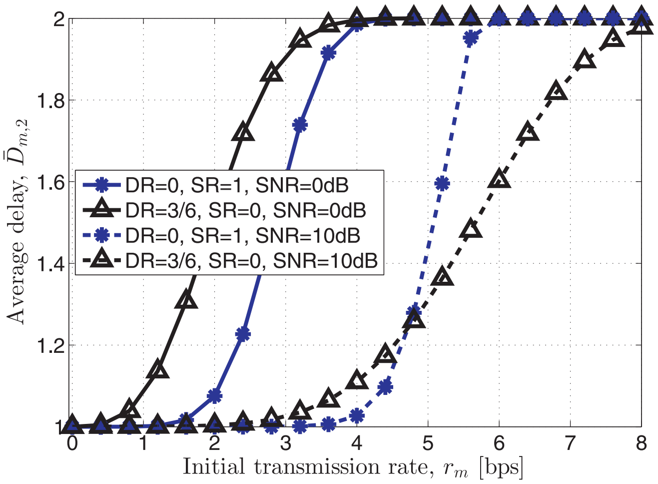

Fig. 6. Illustrates the behavior of average delay, $\overline{D}_{m,T+1}$ in (11) with changes in the initial transmission rate, r_m of the mth user. Here the results are obtained for the case with a maximum of $T = 1$ retransmission, i.e., a maximum of 2 (re)transmission, at SNR of 0 dB and 10 dB.

average delay. Increasing the initial transmission rate, higher SNR of 10 dB is required to achieve the same average delay. The same trend is observed at the bottom of the curve. However, increasing the transmission rate r_m above 5 bps at SNR of 10 dB, the dedicated resources outperform the case of sharing the resources in terms of average delay. This can be explained using (13), where at high SNRs the system tends to be dominated by interference due to sharing the frequency resources. However, in (14), there is no interference and the long term throughput increases with T substantially, due to which the need to retransmit decreases, thereby decreasing the average delay as per (11). Note that the results in Fig. 6 are obtained for a slow fading model, while the same qualitative conclusions hold for the fast fading condition as well.

Interestingly, the figures indicate that, while the HARQ increases the long term throughput and decreases the outage probability considerably, it does not lead to considerable average delay increment for a large range of the initial transmission rates. Therefore, the HARQ can be considered as an efficient approach for the limited feedback CoMP setups.

IV. CONCLUSIONS AND FUTURE WORK

In this paper, we studied the frequency allocation to users with CSIR in a non-coherent JT-CoMP setup with HARQ INR which is very useful in a dense deployment of small cells in 5G communication systems. In terms of the long term throughput and the users' outage probability, we found that at low to medium SNRs sharing the frequency resources is the best way to allocate frequencies to the users, while dedicating disjoint frequencies to the user is better at high SNR. We showed that HARQ is a very efficient scheme for limited feedback in this CoMP setup, resulting in high long term throughput and low outage probability with an affordable average delay. Also, as demonstrated, incorporating HARQ with limited ACK/NACK based feedback performs better in a fast fading environment than in a slow fading environment, in terms of outage probability and the long term throughput. Interestingly the results show that the best long term throughput is observed when sharing the frequency resource at low/medium SNRs, or dedicating the frequency resource at high SNRs. There is no intermediate allocation of frequency for typical SNR values of practical interest as considered in this work. Our proposed scheme substantially simplifies the power allocation design as we do not need adaptive power allocation. Our future work explores the effect of pathloss and shadow fading with the proposed frequency allocation.

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