Wireless Four-Way Relaying using Physical Layer Network Coding with Nested Lattices

Huaping Liu†,∗, Elisabeth de Carvalho∗, Petar Popovski∗ and Yuping Zhao†
†State Key Laboratory of Advanced Optical Communication Systems and Networks, Peking University, China
∗Department of Electronic Systems, Aalborg University, Denmark
Email: {liuhp,yuping.zhao}@pku.edu.cn, {petarp,edc}@es.aau.dk

***Abstract*— Two-way relaying in wireless systems has initiated a large research effort during the past few years. In particular, structured codes and lattices are instrumental for achieving high rates when using Physical Layer Network Coding (PLNC). In a broader perspective, the two-way traffic pattern is rather limited and it is of interest to investigate other traffic patterns where such a simultaneous processing of information flows can bring performance advantage. In this paper we consider a scenario with *four-way relaying*, where each of the two Mobile Stations (MSs) has a two-way connection to the same Base Station (BS), while each connection is through a dedicated Relay Station (RS). The two RSs are in the range of the BS, but they are deployed to have *antipodal positions* within the cell and they do not interfere with each other, i. e. achieve a perfect spatial reuse. We introduce communication schemes for serving the four communication flows in two transmission phases. Each phase consists of combined broadcast and multiple access. The main design ingredients are dirty paper coding nested lattice code codes. We compare the performance with a reference scheme that utilizes Decode-and-Forward (DF). The results show that the usage of structured codes in the four-way relaying scenario can lead to significant enlargement of the achievable rate region.**

I. Introduction

The idea of network coding [1] brings profound gains in a wireless setting, notably in a scenario with two-way relaying. An overview of bidirectional relay protocols is given in [2]. In order to solve the noise accumulation problem of analog network coding (ANC) [3] [4], denoise-and-forward (DNF) has been proposed [5], also known by the name Physical Layer Network coding (PLNC) [6]. In [7], the authors proved that lattice based structured code can achieve point-to-point channel capacity. Paper [8] and [9] apply the idea of structured code to improve PLNC in two-way relay network by using nested lattice code to achieve the cut-set bound within 1/2 bit, which is the best known DNF method.

Although the benefits of wireless network coding have largely been confined to the canonical two–way relaying scenario, one can identify the underlying principles and then apply them in more generalized scenarios. Those principles are (1) simultaneous service of multiple flows over the wireless medium and (2) cancellation of interference based on previously gathered information. We have utilized these principles in order to devise transmission schemes in the case when the flows to a relayed and a direct user are simultaneously served [10]. One of the main objectives of this paper is to leverage on these principles and investigate how they can be applied in a new, and very practical scenario. Namely, this involves *four-way relaying*, in which two Mobile Stations (MSs) have each a two-way connection to the same BS, while each of them uses a dedicated Relay Station (RS). This is depicted on Fig. 1. Both RSs are in the range of the same BS, but they are at the "opposite sides" of the cell (antipodal), in a sense that they do not interfere with each other. A similar scenario has been considered [11], where DS-CDMA is used to avoid interference, the nodes use BPSK modulation and the relay applies PLNC i. e. DNF. Unlike [11], in our work we do not use orthogonal CDMA codes, but take advantage of the antipodal relay deployment in order to coordinate the interference. Furthermore, in our work we are not constrained to a specific modulation type, since we analyze the achievable rate region when the nodes use information-theoretic codebooks.

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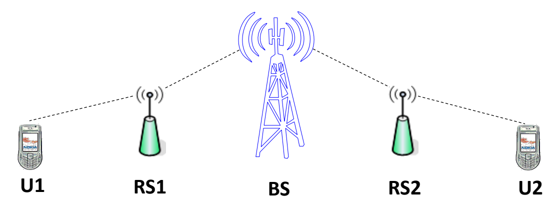

Figure 1. A cellular network with four-way relaying using a Base Station (BS), two Relay Stations (RSs), denoted RS1 and RS2, and two Mobile Stations (MSs) U1 and U2.

Our transmission scheme uses the principles of dirty paper coding and nested lattice codes. Dirty paper coding [12] is capacity–achieving for Gaussian broadcast channels, while nested lattice codes [7] are useful in scenarios in which independent interfering flows should result in a structured output. We derive the achievable rate region of the four-way relaying network [13]. The proposed communication scheme serves the four communication flows in two transmission phases. Each phase consists of combined broadcast and multiple access, integrating the benefits of dirty paper coding and nested lattice codes. The results show that the usage of structured codes in the four-way relaying scenario can lead to significant enlargement of the achievable rate region, when compared to

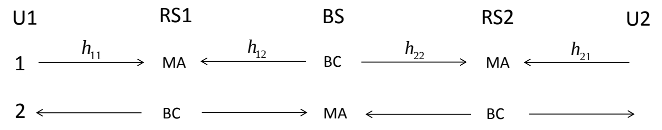

Figure 2. A new transmission scheme for four–way relaying consisting of two phases.

the four way relaying schemes that use Decode-and-Forward, proposed in [13]. However, the performance degrades when the received power at one RS is significantly different from the received power at the other RS.

II. System and Channel Models

Consider a bidirectional cellular network in which a BS intends to exchange information with two mobile stations U1 and U2 with the aid of two relay stations RS1 and RS2, as Fig. 1 shows. All the nodes are half-duplex, such that a node can either transmit or receive at a given time. We assume that the MSs do not have direct links to/from the BS. For simplicity, we use $1, R1, B, R2, 2$ as the indices in the formulas to denote the mobile station U1, relay station RS1, base station BS, relay station RS2 and mobile station U2 respectively. The channels are denoted by h_{11}(U1-RS1), h_{12}(RS1-BS), h_{22}(BS-RS2) and h_{21}(RS2-U2). Each channel $h_l, l \in \{11, 12, 22, 21\}$, is reciprocal, known at all the nodes. Each MS has a two-way, uplink/downlink traffic to/from the BS. Considering that we are using lattices, it is more convenient to work with *real* channels [7], such that all the transmitted signals and the noise are real. Specifically, all receivers observe an i.i.d. Additive White Gaussian Noise (AWGN) with zero mean and unit variance, i. e. $z_j \sim \mathcal{N}(0,1), j \in \{1, R1, B, R2, 2\}$. If node j is transmitting, its transmission power is bounded by $\bar{P}_j, j \in \{1, R1, B, R2, 2\}$, i.e., $E\left\{|x_j|^2\right\} = P_j \le \bar{P}_j$. The capacity of a single link is $C(\gamma) = \frac{1}{2}\log_2(1+\gamma)$, where γ is the Signal-to-Noise Ratio (SNR).

In the proposed scheme, the two downlink signals for U1 and U2 are broadcasted by the BS using dirty paper coding. The whole transmission contains only two phases where communications with the two users occur simultaneously. Furthermore, a MA process and a BC process occur simultaneously in each phase, as the Fig. 2 shows. The two phases have different durations: the duration of the first phase is denoted as τ while the duration of the second phase is denoted as $1-\tau$. The parameter τ is optimized as described in Section IV. Furthermore, the proposed scheme makes full use of the side information at U1, U2 and BS to cancel the self-interference.

III. Achievable rate regions of two-phase four-way relaying

In the proposed scheme, the BS broadcasts two superposed signals:

$$x_B = x_{B1} + x_{B2}, \tag{1}$$

where x_{B1} and x_{B2} carry the signals intended for U1 and U2 and they are encoded using dirty paper coding, as described in Section III-A1. The total transmit power is P_B is split as $E\left\{|x_{B1}|^2\right\} = \beta P_B$, $E\left\{|x_{B2}|^2\right\} = (1-\beta) P_B$ where the parameter β is optimized as described in Section IV. The signals sent by U1 and U2 are denoted by x_1 and x_2, respectively. We define R_i^d as the downlink data rate of Ui and R_i^u as the uplink data rate of Ui.

At the BS, both signals x_{B1} and x_{B2} are designed so that the interference (i.e. the signal intended to U2) at the receiver RS1 is cancelled out. The interference at RS2 (i.e. the signal intended to U1) is treated as noise. This strategy corresponds to a 4-dimensional achievable rate region denoted as $\mathfrak{R}_1$ and shown in III-E. Likewise, the symmetric encoding strategy can be adopted where the interference is cancelled out at RS2 while RS1 treats the interference as noise, corresponding to an achievable rate region $\mathfrak{R}_2$.

Proposition 1: The achievable rate region of the two-phase four-way relaying scheme using lattice DNF is the convex closure of all 4-dimensional rate tuples satisfying $\left(R_1^u, R_1^d, R_2^u, R_2^d\right) \in \mathfrak{R}_1 \cup \mathfrak{R}_2$.

In the following, we describe how to obtain the capacity region $\mathfrak{R}_1$. By symmetry, identical approach can be used to derive $\mathfrak{R}_2$.

A. Encoding in the first phase

1) Dirty paper encoding at BS: We denote a lattice $\Lambda_k \in \mathbb{R}^n$, where n is the lattice code dimension, $k \in \{1, 2, B1, B2, c\}$. The fundamental Voronoi region of Λ_k is denoted by $\mathcal{V}_k$. The lattice-based dirty paper codes at BS can be described as follows:

$$x_{B1} = \frac{1}{h_{12}}[w_{B1} - \alpha h_{12} x_{B2} - V_{B1}] \bmod \Lambda_{B1} \tag{2a}$$

$$x_{B2} = \frac{1}{h_{22}}[w_{B2} - V_{B2}] \bmod \Lambda_{B2} \tag{2b}$$

where w_{Bi} is the message for Ui and uniformly distributed over $\mathcal{V}_{Bi}$. Furthermore, the message belongs to a codebook C_{Bi}, which is defined through nested lattices: a coarse lattice Λ_{Bi} and the fine lattice Λ_c, such that we can write $C_{Bi} = \{\Lambda_c \bmod \Lambda_{Bi}\}$. We use the mod operation as defined in [7]. The dither signal is denoted by V_{Bi}; it is uniformly distributed over $\mathcal{V}_{Bi}$ and known at each node. The dither signal is introduced to make sure that x_{Bi} is independent from w_{Bi} which is proved in [7, Lemma 1]. We adopt a MMSE design as in [14, Eq.4.28,4.29]. α is the MMSE coefficient applied at the receiver. To ensure that the interference is cancelled out of RS1, the transmitter performs a scaling by α as described in equation (2a). Finally, in order to satisfy the power constraint, the variances of the lattice codes are set as follow: $\mathrm{Var}(\Lambda_{B1}) = \beta P_B h_{12}^2$, $\mathrm{Var}(\Lambda_{B2}) = (1-\beta) P_B h_{22}^2$.

2) Lattice encoding at Ui: User i encodes the codeword as follow:

$$x_i = \frac{1}{h_{i1}}[w_i - V_i] \bmod \Lambda_i. \tag{3}$$

where $w_i \in C_i = \{\Lambda_c \bmod \Lambda_i\}$ is the message sent from Ui uniformly distributed over $\mathcal{V}_i$. Similarly, V_i is the dither signal uniformly distributed over $\mathcal{V}_i$ and known at each node. The

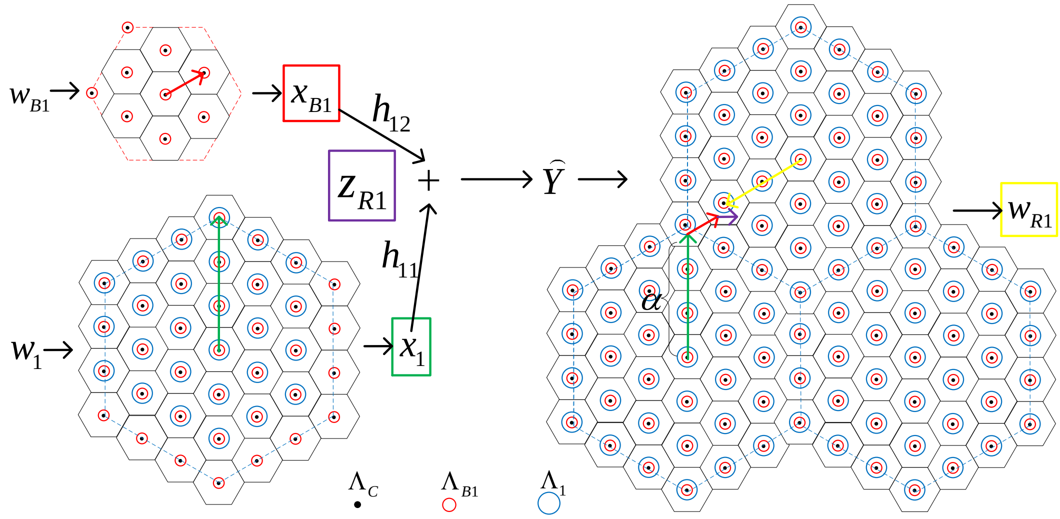

Figure 3. Case 1 at RS1, $\Lambda_1 \subseteq \Lambda_{B1} \subseteq \Lambda_C$. When denoise decoding, the receiver observes a noisy sum of the transmitted vectors and determines the closest lattice point based on Λ_1. In this figure we assume $h_{11} = h_{12} = 1$.

variance of the lattice is set to $\mathrm{Var}(\Lambda_i) = P_i {h_{i1}}^2$ to satisfy the transmit power constraints.

3) Nested lattice constraint: For a proper decoding, the lattice codewords sent from the BS and Ui have to satisfy the nested lattice constraint which states the following: if $\mathrm{Var}(\Lambda_{Bi}) \geq \mathrm{Var}(\Lambda_i)$, then $\Lambda_{Bi} \subseteq \Lambda_i \subseteq \Lambda_c$, otherwise $\Lambda_i \subseteq \Lambda_{Bi} \subseteq \Lambda_c, i = 1, 2$. The existence of the nested lattice codewords is guaranteed by [15, Theorem 2]. Λ_i and Λ_{Bi} are assumed to be simultaneously Rogers-good and Poltyrev-good, while Λ_c is Poltyrev-good [7]. Note that we do not assume a nested relationship between lattices $\{\Lambda_{B1}, \Lambda_1\}$ and $\{\Lambda_{B2}, \Lambda_2\}$.

B. Decoding at RS

At the end of phase 1, RSi receives

$$y_{Ri} = h_{i1}x_i + h_{i2}x_B + z_{Ri}, i = 1, 2. \tag{4}$$

where x_B is defined in (1).

1) Decoding at RS1: The transmission power and the channel gains decide on the nested relation between lattices Λ_{B1} and Λ_1 introduced in section III-A3. Two cases can be distinguished: $\Lambda_1 \subseteq \Lambda_{B1}$ and $\Lambda_{B1} \subseteq \Lambda_1$.

Case 1: $\Lambda_1 \subseteq \Lambda_{B1}$, the decoding is based on the coarse lattice Λ_1 as shown in Fig. 3. An MMSE receiver is considered where decoding is based on αy_{R1} and more precisely on the following modulo lattice additive (MLAN) channel signal:

$$\widehat{Y} = [\alpha y_{R1} + V_{B1} + V_1] \bmod \Lambda_1 \tag{5}$$

To easily exhibit the desired signal and the effective noise term, we rely on the following decomposition (see [15, Eq.10] and [16, Eq.74]):

$$\begin{aligned}\widehat{Y} = [&h_{11}x_1 + h_{12}x_{B1} + \alpha y_{R1} + V_{B1} + V_1 \\ &- h_{11}x_1 - h_{12}x_{B1}] \bmod \Lambda_1\end{aligned} \tag{6}$$

Using (4), we get:

$$\begin{aligned}\widehat{Y} = [&h_{11}x_1 + h_{12}x_{B1} + \alpha h_{12}x_{B2} + V_{B1} + V_1 \\ &+ (\alpha - 1)(h_{11}x_1 + h_{12}x_{B1}) + \alpha z_{R1}] \bmod \Lambda_1\end{aligned} \tag{7}$$

Based on [7, Eq.8], we have $x \bmod \Lambda_k = x - Q_{\Lambda_k}(x)$ for all $x \in \mathbb{R}^n$, here $Q_{\Lambda_k}(x)$ is a nearest neighbor of x in Λ_k. Then (7) can be simplified as

$$\begin{aligned}\widehat{Y} = [&w_1 + w_{B1} - Q_{\Lambda_1}(w_1 - V_1) \\ &- Q_{\Lambda_{B1}}(w_{B1} - \alpha h_{12}x_{B2} - V_{B1}) \\ &+ (\alpha - 1)(h_{11}x_1 + h_{12}x_{B1}) + \alpha z_{R1}] \bmod \Lambda_1.\end{aligned} \tag{8}$$

Finally, noting that $Q_{\Lambda_k}(x) \bmod \Lambda_k = 0$ for all $x \in \mathbb{R}^n$, we get

$$\begin{aligned}\widehat{Y} = [&w_1 + w_{B1} - Q_{\Lambda_{B1}}(w_{B1} - \alpha h_{12}x_{B2} - V_{B1}) \\ &+ (\alpha - 1)(h_{11}x_1 + h_{12}x_{B1}) + \alpha z_{R1}] \bmod \Lambda_1\end{aligned}. \tag{9}$$

Strictly speaking, w_1 and w_{B1} are the desired signals. However, the signal $Q_{\Lambda_{B1}}(w_{B1} - \alpha h_{12}x_{B2} - V_{B1})$ is not considered as noise and will be forwarded by the RS to the BS and user 1 as this signal can be cancelled out at both destinations. Indeed, BS knows a priori all the signals in $Q_{\Lambda_{B1}}(w_{B1} - \alpha h_{12}x_{B2} - V_{B1})$ and $Q_{\Lambda_{B1}}(w_{B1} - \alpha h_{12}x_{B2} - V_{B1}) \bmod \Lambda_{B1} = 0$, so that U1 also can cancel this term by performing a modulo operation as we explain in III-D.

Next, we denote as w_{R1} the desired signal and $\widehat{z}_{R1}$ the effective noise:

$$w_{R1} = [w_1 + w_{B1} - Q_{\Lambda_{B1}}(w_{B1} - \alpha h_{12}x_{B2} - V_{B1})] \bmod \Lambda_1. \tag{10}$$

$$\widehat{z}_{R1} = (\alpha - 1)(h_{11}x_1 + h_{12}x_{B1}) + \alpha z_{R1}. \tag{11}$$

α is determined to minimize the mean squared error $E\left\|\widehat{z}_{R1}\right\|^2$ and has for expression:

$$\alpha = \frac{\mathrm{Var}(\Lambda_1) + \mathrm{Var}(\Lambda_{B1})}{\mathrm{Var}(\Lambda_1) + \mathrm{Var}(\Lambda_{B1}) + 1}. \tag{12}$$

The resulting variance of the effective noise is:

$$E\left\|\widehat{z}_{R1}\right\|^2=\frac{\text{Var}\left(\Lambda_1\right)+\text{Var}\left(\Lambda_{\text{B}1}\right)}{\text{Var}\left(\Lambda_1\right)+\text{Var}\left(\Lambda_{\text{B}1}\right)+1}. \quad (13)$$

Refering to [16, Eq.63], the corresponding maximal achievable rate is $\log_2\left(\frac{\text{Var}(\Lambda_1)}{E\left\|\widehat{z}_{R1}\right\|^2}\right)$. Then considering the first phase transmission duration τ, we get

$$R_1^u<\frac{\tau}{2}\left[\log_2\left(\frac{{h_{11}}^2P_1}{{h_{11}}^2P_1+\beta P_B{h_{12}}^2}+{h_{11}}^2P_1\right)\right]^+. \quad (14)$$

where $[x]^+\triangleq\max\{x,0\}$.

Because $\Lambda_1\subseteq\Lambda_{B1}$ and based on [8, section IV.A], we have $R_1^u=R_1^d+\log_2\left(\frac{\text{Var}(\Lambda_1)}{\text{Var}(\Lambda_{B1})}\right)$. Substituting the above equation into (14), we obtain

$$R_1^d<\frac{\tau}{2}\left[\log_2\left(\frac{\beta P_B{h_{12}}^2}{{h_{11}}^2P_1+\beta P_B{h_{12}}^2}+\beta P_B{h_{12}}^2\right)\right]^+. \quad (15)$$

Case 2: $\Lambda_{B1}\subseteq\Lambda_1$, the decoding is based on the coarse lattice Λ_{B1}. Adopting a similar decoding approach as in case 1, we can obtain the rate constraints for R_1^u and R_1^d. Coincidently, (14) and (15) are also the rate constraints for R_1^u and R_1^d in case 2. However, the denoised message is different and is equal to:

$$w_{R1}=\left[w_1+w_{B1}-Q_{\Lambda_1}\left(w_1-V_1\right)\right]\bmod\Lambda_{B1}. \quad (16)$$

2) Decoding at RS2: At RS2, the interference is not cancelled but is treated as noise. Decoding is carried out from the received signal: From (4), we get

$$y_{R2}=h_{21}x_2+h_{22}x_{B2}+\left[h_{22}x_{B1}+z_{R2}\right] \quad (17)$$

where $h_{22}x_{B1}+z_{R2}$ is the effective noise. Note that Λ_2 and Λ_{B2} have a nested relationship. However, Λ_{B1} is independent from Λ_2 and Λ_{B2}. x_{B1} is encoded based on lattice Λ_{B1} and cannot be properly decoded using lattice Λ_2 or Λ_{B2}. Therefore, x_{B1} is considered as noise in the decoding process.

x_2 and x_{B2} contain the 2 signals to be decoded at RS2. When Λ_{B1} is Rogers-good as seen in section III-A3, x_{B1} approaches a Gaussian random variable as $n\to\infty$ [7]. Then the effective noise in (17) can be viewed as Additive White Gaussian Noise. This situation is the same as [8] where there are two signals to be decoded. From [8, Eq.9], we get the rate constraints as follow:

$$R_2^u<\frac{\tau}{2}\left[\log_2\left(\frac{{h_{21}}^2P_2}{{h_{21}}^2P_2+{h_{22}}^2\left(1-\beta\right)P_B}+\frac{{h_{21}}^2P_2}{\beta P_B{h_{22}}^2+1}\right)\right]^+. \quad (18)$$

$$R_2^d<\frac{\tau}{2}\left[\log_2\left(\frac{{h_{22}}^2\left(1-\beta\right)P_B}{{h_{21}}^2P_2+{h_{22}}^2\left(1-\beta\right)P_B}+\frac{{h_{22}}^2\left(1-\beta\right)P_B}{\beta P_B{h_{22}}^2+1}\right)\right]^+. \quad (19)$$

If $\Lambda_2\subseteq\Lambda_{B2}$, the denoised message in RS2 is $w_{R2}=[w_2+w_{B2}-Q_{\Lambda_{B2}}\left(w_{B2}-V_{B2}\right)]\bmod\Lambda_2$. Otherwise, the denoised message in RS2 is $w_{R2}=\left[w_2+w_{B2}-Q_{\Lambda_2}\left(w_2-V_2\right)\right]\bmod\Lambda_{B2}$.

C. Re-encoding at RSi

RSi re-encodes w_{Ri} using a Gaussian code as x_{Ri} then broadcasts it.

D. Decoding in the second phase

U1 knows w_1 a priori and decodes x_{Ri} to get w_{R1}. If $\Lambda_1\subseteq\Lambda_{B1}$, then $[x\bmod\Lambda_1]\bmod\Lambda_{B1}=x\bmod\Lambda_{B1}$. In this case, $w_{R1}=\left[w_1+w_{B1}-Q_{\Lambda_{B1}}\left(w_{B1}-\alpha h_{12}x_{B2}-V_{B1}\right)\right]\bmod\Lambda_1$. U1 can get w_{B1} by $w_{B1}=[w_{R1}-w_1]\bmod\Lambda_{B1}$.

If $\Lambda_{B1}\subseteq\Lambda_1$, then $[x\bmod\Lambda_{B1}]\bmod\Lambda_1=x\bmod\Lambda_1$. In this case, $w_{R1}=\left[w_1+w_{B1}-Q_{\Lambda_1}\left(w_1-V_1\right)\right]\bmod\Lambda_{B1}$. U1 can get w_{B1} by $w_{B1}=[w_{R1}-w_1]\bmod\Lambda_1$.

So for U1, successfully obtaining w_{R1} will lead to successfully obtaining w_{B1}. Similarly, for U2, successfully obtaining w_{R2} will lead to successfully obtaining w_{B2}. Also similar to the above two cases, at BS if w_{R1} and w_{R2} are successfully obtained, w_1 and w_2 can be obtained. Summarizing, if w_{Ri} can be obtained correctly, each node can get the wanted message.

U1, U2 and BS independently decode x_{Ri} to get w_{Ri}. The successful decoding of x_{R1} by U1 is conditioned to (20a). The successful decoding of x_{R2} by U2 is conditioned to (20b). The successful decoding of x_{R1} and x_{R2} by BS is conditioned to (20c), (20d) and (20e). Note that $1-\tau$ is the duration of the second phase.

$$\begin{cases} R_1^d<\left(1-\tau\right)C\left({h_{11}}^2P_{R1}\right) & \text{(20a)}\\ R_2^d<\left(1-\tau\right)C\left({h_{21}}^2P_{R2}\right) & \text{(20b)}\\ R_1^u<\left(1-\tau\right)C\left({h_{12}}^2P_{R1}\right) & \text{(20c)}\\ R_2^u<\left(1-\tau\right)C\left({h_{22}}^2P_{R2}\right) & \text{(20d)}\\ R_1^u+R_2^u<\left(1-\tau\right)C\left({h_{12}}^2P_{R1}+{h_{22}}^2P_{R2}\right) & \text{(20e)}\end{cases}$$

E. Rate region

As a conclusion, the rate region $\mathfrak{R}_1$ is defined as: $\mathfrak{R}_1=\left\{\left(R_1^u,R_1^d,R_2^u,R_2^d\right)\mid\left(R_1^u,R_1^d,R_2^u,R_2^d\right)\in(14),(15),(18),(19),(20)\right\}$.

IV. Achievable rates with fixed downlink-uplink rate ratio

The achievable rate regions described in section III have four dimensions. In order to get a better insight into the achievable rates, we impose a ratio between the uplink and downlink rates. It is practical to fix the downlink-uplink rate ratio for a given type of application e. g. gaming and calls have ratio of 1:1, web browsing has a ratio of about 5:1 [17]. We assume that, for the $i-$th user, the downlink rate demand R_i^d is related to the uplink rate demand R_i^u, as $R_i^d=\theta_iR_i^u$, see [18]. Applying the downlink-uplink rate ratio, the dimension of the achievable rate region degrades to 2. In this section, for simplicity, we plot the achievable rate regions of the rate pair $\left(R_1^u,R_2^u\right)$ and assume each node has equal transmission power which is $P_j=10, j\in\{1,R1,B,R2,2\}$.

The achievable rate region of the two-phase DNF relaying scheme is related to the time ratio τ and superposition ratio β. Those two parameters need to be optimized to get the envelope of the achievable rate region. In this optimization, we fix the rate R_1^u of user 1 to a set value r_1 and find the maximal rate R_2^u

of user 2. The parameter r_1 describes the feasible rate values for user 1, $r_1 \in [0, R^u_{1\,\max}]$ where $R^u_{1\,\max}$ is determined via another optimization. The achievable rate regions of the two-phase scheme can be obtained by a two-step optimization. The first step is to find $R^u_{1\,\max} = \max\{R^u_1\}$, as shown below:

$$\max_{0\leqslant\tau\leqslant 1, 0\leqslant\beta\leqslant 1} \quad R^u_1, \quad s.t.\ \mathfrak{R}_1 \cup \mathfrak{R}_2 \text{ and } R^u_2 = 0, \text{ given } \theta_1, \theta_2. \tag{21}$$

Then, finding the achievable rate region is equivalently to solve the following optimization problem for each feasible rate point $r_1 \in [0, R^u_{1\,\max}]$:

$$\max_{0\leqslant\tau\leqslant 1, 0\leqslant\beta\leqslant 1} \quad R^u_2, \quad s.t.\ \mathfrak{R}_1 \cup \mathfrak{R}_2 \text{ and } R^u_1 = r_1, \text{ given } \theta_1, \theta_2. \tag{22}$$

The formulas (21) and (22) are two conventional constrained nonlinear optimization problems for which a numerical solution for the achievable rate region can be found by sequential quadratic programming.

V. Numerical results

The achievable rate regions of rate pair (R^u_1, R^u_2) are presented in Fig. 4 and Fig. 5. We compare the region obtained by lattice based DNF to a simple DF strategy as described in [13].

When the data rate is symmetric, $\theta_1 = \theta_2 = 1$, we focus on the performance under different SNR conditions as Fig. 4 shows. We consider three cases. The first case (S1) is when the links have the same SNR, that is ${h_{11}}^2 = {h_{12}}^2 = {h_{22}}^2 = {h_{21}}^2 = 10$. The second case (S2) is when the links that are direct to the BS have a lower SNR than the links that are direct to the users, that is ${h_{11}}^2 = {h_{21}}^2 = 10, {h_{12}}^2 = {h_{22}}^2 = 1$. The third case (S3) is when the links that are direct to the BS have a higher SNR than the links that are direct to the users, that is ${h_{11}}^2 = {h_{21}}^2 = 1, {h_{12}}^2 = {h_{22}}^2 = 10$.

When the data rate is asymmetric, $\theta_1 \neq 1, \theta_2 \neq 1$, we fix the SNR as ${h_{11}}^2 = {h_{12}}^2 = {h_{22}}^2 = {h_{21}}^2 = 10$ and focus on the performance under different downlink-uplink rate ratio configurations as Fig. 5 shows. We again consider three cases. The first case (S4) is when the downlink date rate is smaller than the uplink date rate for both U1 and U2, that is $\theta_1 = \theta_2 = 0.5$. The second case (S5) is when the downlink date rate is larger than the uplink date rate for both U1 and U2, that is $\theta_1 = \theta_2 = 2$. The third case (S6) is the downlink data rate is larger than the uplink date rate for U1 and the downlink data rate is smaller than the uplink date rate for U2, that is $\theta_1 = 2$, $\theta_2 = 0.5$.

When ${h_{11}}^2 = {h_{12}}^2 = {h_{22}}^2 = {h_{21}}^2 = 10$, the lattice DNF has larger rate region than the DF method. When the receiving powers at RS are far from each other as s2 and s3 show, lattice DNF is worse than DF. The above phenomenon comes from the fact that lattice decoding is performed based on the lattice with larger power. Hence, in general, the decoding of the signal with lower power will have poor performance. When the receiving powers at RS are similar, lattice based decoding performance is good.

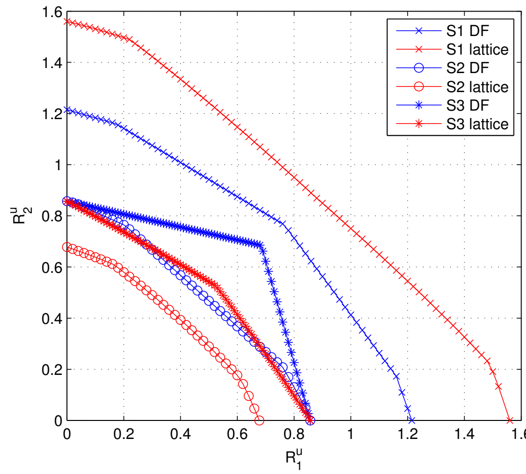

Figure 4. Comparison the rate regions of the rate pair (R^u_1, R^u_2) with $\theta_1 = \theta_2 = 1$

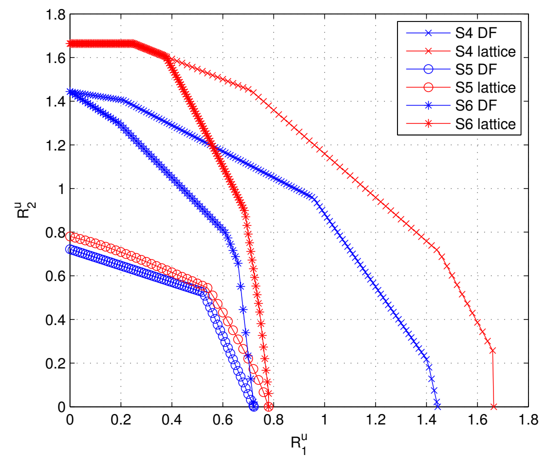

Figure 5. Comparison the rate regions of the rate pair (R^u_1, R^u_2) with ${h_{11}}^2 = {h_{12}}^2 = {h_{22}}^2 = {h_{21}}^2 = 10$

VI. Conclusion

In this paper we have considered the communication scenario of four-way relaying, which is a generalization of the two-way relaying scenario, commonly used to apply the principles of wireless network coding. We have devised a new communication scheme that leverages on the idea of nested lattices for physical layer network coding and the principles of dirty paper coding. The scheme consists of two phases, multiple access and a broadcast, respectively. The numerical results show that the achievable rate region is significantly increased compared to the communication schemes that utilize Decode-and-Forward (DF). In our future work we will relax the assumption on the ideal spatial reuse for the Relay Stations (RSs) and devise schemes that account the interference that occurs between the RSs.

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