Application of a Leakage Based Precoding Scheme to Mitigate Intrinsic Interference in FBMC

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***Abstract*—Orthogonal frequency division multiplexing with offset QAM (OFDM/OQAM) modulation scheme, more commonly known as filter bank multicarrier (FBMC), has received greater attention recently, due to its spectral efficiency compared to standard OFDM. However, intrinsic interference in FBMC causes a negative effect on the system performance depending on the channel condition. Hence, in this paper, we analyze the interference in OFDM/OQAM systems and introduce a precoder based on signal-to-leakage-plus-noise ratio (SLNR) to overcome the effect of interference on the system. First we implement the system in a more efficient way using inverse fast Fourier transform (IFFT). Then based on that model, the precoding matrix is generated. Finally through simulation and analysis, reduction of interference and improvement of performance are investigated.**

***Index Terms*—FBMC, OFDM/OQAM, MCM, IFFT, SINR, SLNR, SNR, Equilization, BER.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) is the most prominent multi-carrier modulation (MCM) used in modern wireless communication networks due to its comparatively high performance against narrow-band interference and simplicity of transceivers [1]. However, in OFDM, some redundant data, cyclic prefix (CP), is inserted between data symbols in order to maintain the orthogonality among subcarriers and reduce the inter carrier interference (ICI) [2]. This leads to a loss of spectral efficiency. In addition, large side lobes in the spectrum generated by the rectangular pulse may introduce a significant amount of ICI to the system due to non ideal synchronization.

As a solution to these imperfections, filter bank multicarrier modulation (FBMC) was proposed as an alternative to OFDM. One significant difference in FBMC is, how it maintains the orthogonality between subcarriers without a CP, as in OFDM. The technique is to use two types of orthogonal filter banks known as $sine$ and $cosine$ which are generated from a prototype filter investigated in [3]. The pulse shape of this prototype filter is different from the rectangular window used in OFDM and more localized in both frequency and time domain, hence out of band frequency leakage is minimized.

In addition, each subcarrier is modulated with the offset quadrature amplitude modulation (OQAM). The basic difference in OQAM compared to standard QAM is transmitting symbols as real and imaginary samples and the offset of half of the symbol period between them [4]. This results in a better modulation technique which provides high spectral efficiency with respect to OFDM.

However, in FBMC, using real part of the received signal is sufficient to recover the original data symbols [5]. Thus, the remaining imaginary part appears as an intrinsic interference term when the channel is frequency selective. Although the data is always orthogonal to this interference term, latter becomes a source of problems when combining FBMC with other systems like MIMO and CR [6]. Therefore careful removal or compensation of these interference components is vital for better performance gain.

Previous work

The total interference due to inter-symbol interference (ISI) and ICI in FBMC has been analyzed in [5] and closed form expressions have been obtained. Furthermore, a interference mitigating technique based on the Alamouti coding scheme and maximum likelihood detection (MLD) is presented in [6]. There, they have taken the advantage of combining FBMC with MIMO technique and its diversity gain to improve the performance of the system. On the other hand, [7] has introduced an efficient scheme based on spatial diversity to cancel the ISI and ICI in OFDM/OQAM. Also here they have shown how the multiuser gain can be used to enhance the system performance compared to the Alamouti technique.

In addition, considering the complexity of the receiver as its resources are limited depending on the requirement, a precoding scheme based on signal-to-leakage-plus-noise ratio (SLNR) is implemented in [8] and [9] which suppresses the ISI/ICI in MIMO-OFDM. In [10] and [11] authors have investigated how the precoding matrix can be generated based on zero forcing (ZF) and minimum mean square error (MMSE) equalization respectively.

Our contribution

Here we consider the effect on interference in FBMC with the introduction of the precoder based on SLNR at the transmitter side. First, a feasible implementation is obtained using the fast Fourier transform and with the help of polyphase components such that precoding can be carried out for each carrier independently. Then based on that model, simulation is carried out for the proposed as well as the existing OFDM systems with different equalization methods and finally, improvement of the performance is discussed.

The remainder of this paper is organized as follows. In Section II the continuous and discrete time implementations and the FFT implementation method is presented. Section III gives detailed design of the precoder based on SLNR. Implementation of time domain equalizers is introduced in section IV. Simulation results are then presented and discussed in section V and the last section concludes the paper.

II. System Implementation

Let us consider an FBMC scheme with $2M$ subcarriers and $1/T$ symbol rate. Then the transmitted FBMC signal can be written in the following manner [5], assuming $2M$ symbols are transmitted during T,

$$s[k] = \sum_{\acute{m}=0}^{2M-1} \sum_{\acute{n}=-\infty}^{\infty} d_{\acute{m},\acute{n}} h_{\acute{m},\acute{n}}[k] \tag{1}$$

where $d_{\acute{m},\acute{n}}(\acute{n}\epsilon\mathbb{Z}, \acute{m} = 0, 1, ..., 2M-1)$ denotes the data signals conveyed by the sub carrier at index $\acute{m}$ during the time index $\acute{n}$, $s[k]$ is the transmitted FBMC signal, and $h_{\acute{m},\acute{n}}$ represents bank of bandpass finite impulse response (FIR) filters generated from prototype filter in the following way [5].

$$h_{\acute{m},\acute{n}}[k] = p[k - \acute{n}M] \exp\left(\jmath \frac{2\pi}{2M} \acute{m}k\right) \exp(\jmath \phi_{\acute{m},\acute{n}}) \tag{2}$$

where $\phi_{\acute{m},\acute{n}} = \frac{\pi}{2}(\acute{n} + \acute{m}) - \pi\acute{n}\acute{m}$ and $P[.]$ is the prototype filter introduced in [3]. The time domain coefficients of the filter can obtained as below,

$$P[i] = 1 + 2\sum_{k=1}^{K-1} (-1)^k H_k \cos\left(2\pi \frac{ki}{KM}\right) \tag{3}$$

where $1 \leqslant i \leqslant L-1$, $L = KM$ and $P_0 = 0$. Note that L is prototype filter length and K is the overlapping factor.

After that, FBMC signal in (1) is passed through the channel and additive complex valued zero mean white Gaussian noise $w_{m,n}$ with variance σ^2 is added to the transmitted signal. Then signal at receiver input can be formulated as,

$$r[k] = \sum_{l=0}^{L-1} c_l s[k-l] + w_{m,n} \tag{4}$$

where $r[k]$ is the signal at the receiver input and c_l is the equivalent discrete time channel impulse response.

At the receiver side, a bank of FIR filters, $g_{m,n}$ is used to filter the information from each sub channel. Then the estimated signal can be given as [5],

$$\hat{d}_{m,n} = \Re\left\{\sum_{k=-\infty}^{\infty} r[k] g_{m,n}[k]\right\} \tag{5}$$

where $g_{m,n}[k]$ is the analysis filter bank (AFB) generated as in equation (6),

$$g_{m,n}[k] = p[k - nM] \exp\left(-\jmath \frac{2\pi}{2M} mk\right) \exp(-\jmath\phi_{m,n}) s[k] \tag{6}$$

TABLE I

2D Interference Table : Time-Frequency Response

		Time				
		n-2	n-1	n	n+1	n+2
Frequency	m-2	0	0	0	0	0
	m-1	$0.125\jmath$	$0.206\jmath$	$0.239\jmath$	$0.206\jmath$	$0.125\jmath$
	m	0	$-0.564\jmath$	**1**	$-0.564\jmath$	0
	m+1	$0.125\jmath$	$0.206\jmath$	$0.239\jmath$	$0.206\jmath$	$0.125\jmath$
	m+2	0	0	0	0	0

Then substituting equations (1) and (4) into (5), estimated signal $\hat{d}_{m,n}$ can be further simplified as,

$$\begin{aligned} \hat{d}_{m,n} &= \sum_{k=-\infty}^{\infty} \sum_{l=0}^{L-1} c_l s[k-l] g_{m,n}[k] \\ &\quad + \sum_{k=-\infty}^{\infty} w_{m,n} g_{m,n}[k] \\ &= \sum_{k=-\infty}^{\infty} \sum_{\acute{m}=0}^{2M-1} \sum_{\acute{n}=-\infty}^{\infty} \psi_{m,n} d_{\acute{m},\acute{n}} \\ &\quad \times h_{\acute{m},\acute{n}}[k-l] g_{m,n}[k] + \sum_{k=-\infty}^{\infty} w_{m,n} g_{m,n}[k] \end{aligned} \tag{7}$$

where $\psi_{m,n}$ is the 2M-point FFT of the channel impulse response as in (8),

$$\psi_{m,n} = \sum_{l=0}^{L-1} c_l \exp\left(-\jmath \frac{2\pi}{2M} ml\right) \tag{8}$$

Then taking the fast fourier transform and polyphase components of the FIR filter as in [12], [13], an efficient implementation of the transceiver can be derived as shown in Fig. 1, where h_m, $s_{m,n}[k]$, $r_{m,n}[k]$ and g_m represent the polyphase components of synthesis filter bank (SFB), FBMC composite signal, received FBMC signal and analysis filter bank (AFB) respectively.

III. Precoding Scheme for FBMC

In this section, we design the proposed precoder for FBMC which was first discussed for multiuser MIMO in [9], in order to mitigate 2D interference considering 2M subcarriers.

For example, let us consider the design in Fig. 1 in section II, where simulation parameters are set as below,

- Filter length, L=64
- Overlapping factor, K=4
- Number of Carriers, $\acute{M}$=16
- Symbols per Carrier, N=16

Then this 2D interference components associated with the symbol at m^{th} subcarrier and n^{th} time index under distortion free channel can be seen as in TABLE I.

Here we can observe that the interference components adjacent to the symbol which was transmitted, are imaginary with relatively large magnitude. This may make a significant effect on the system performance especially when the channel is frequency selective. Therefore to cope with this problem, interference mitigation techniques are vital with FBMC to obtain the real benefit of this modulation scheme.

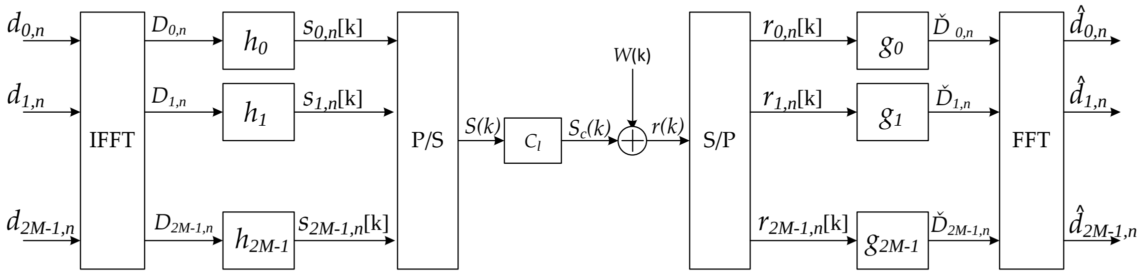

Fig. 1. A block diagram of FBMC transceiver

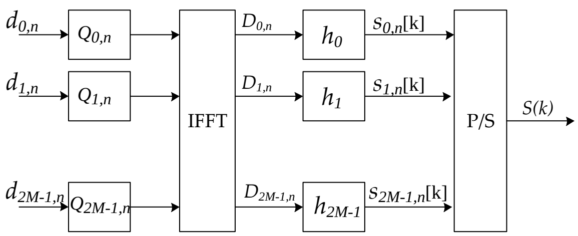

Fig. 2. Block diagram of transmitter with precoder.

Therefore here we use SLNR criterion based precorder solution as despicted in Fig. 2. The main advantage of calculating precoder based on SLNR is, its ability to eliminate intrinsic interference as well as effect caused by channel noise. Also this leads to a closed form solution for the optimization problem defined in (17).

According to scheme shown in Fig. 2, the equation (7) can be extended by inserting the precoder just before the IFFT block as below,

$$\begin{aligned} \hat{d}_{m,n} &= \sum_{k=-\infty}^{\infty} \sum_{\acute{m}=0}^{2M-1} \sum_{\acute{n}=-\infty}^{\infty} \psi_{m,n} Q_{m,n} d_{\acute{m},\acute{n}} \\ &\quad \times h_{\acute{m},\acute{n}}[k-l] g_{m,n}[k] + \sum_{k=-\infty}^{\infty} w_{m,n} g_{m,n}[k] \end{aligned} \tag{9}$$

where $\mathbf{Q}$ is the precoding matrix with dimensions $2M \times N$ and $w_{m,n}$ is AWGN noise.

Here the objective is to recover a particular data symbol at a given subcarrier (m) and time index (n). Hence the other components which are at subcarrier $(\acute{m})$ and time indexes $(\acute{n})$ behave as intrinsic interference terms for this symbol as given below.

$$\begin{aligned} \hat{d}_{m,n} &= \psi_{m,n} Q_{m,n} d_{m,n} \\ &\quad + \underbrace{\sum_{k=-\infty}^{\infty} \sum_{\substack{\acute{m}, m \neq \acute{m} \\ \acute{n}, n \neq \acute{n}}} \psi_{\acute{m},\acute{n}} Q_{\acute{m},\acute{n}} d_{\acute{m},\acute{n}} h_{\acute{m},\acute{n}}[k] g_{\acute{m},\acute{n}}[k]}_{I_{ICI,ISI}} \\ &\quad + \sum_{k=-\infty}^{\infty} w_{m,n} g_{m,n}[k] \end{aligned} \tag{10}$$

where the second term represents the 2D interference caused by adjacent subcarriers and symbols. With respect to the equation (10) signal-to-interference-plus-noise ratio (SINR) of the received signal can be calculated as follows,

$$SINR_{m,n} = \frac{E|\psi_{m,n} Q_{m,n} d_{m,n}|^2}{2M\sigma^2 + E|I_{ICI,ISI}|^2} \tag{11}$$

where, expectation is given by $E[.]$.

However, solving this equation for $Q(.)$ in order to find maximum SINR is challenging and it does not converge to a close form result as discussed in [14]. Therefore, here we use an alternative criterion based on SLNR instead of using SINR.

Starting from the equation (10) the power of desired signal component at subcarrier (m) and time indexes (n) can be calculated as $E|\psi_{m,n} Q_{m,n} d_{m,n}|^2$ and power of interference that is caused by the desired signal on other symbols can be calculated as $2M\sigma^2 + E|\acute{I}_{ICI,ISI}|^2$ where,

$$\acute{I}_{ICI,ISI} = \sum_{\substack{\acute{m}, m \neq \acute{m} \\ \acute{n}, n \neq \acute{n}}} \psi_{\acute{m},\acute{n}} Q_{m,n} d_{\acute{m},\acute{n}} h_{\acute{m},\acute{n}}[k-l] g_{\acute{m},\acute{n}}[k] \tag{12}$$

With this argument we define the SLNR of the symbol at subcarrier (m) and time index (n) as,

$$SLNR_{m,n} = \frac{E|\psi_{m,n} Q_{m,n} d_{m,n}|^2}{2M\sigma^2 + E|\acute{I}_{ICI,ISI}|^2} \tag{13}$$

Note that $d_{m,n}$, $Q_{m,n}$, and prototype filter P introduced in (3), are assumed to be normalized as follows,

$$\begin{aligned} & E|d_{m,n}|^2 = 1, \|Q_{m,n}\|^2 = 1 \\ & \sum_{i=0}^{L-1} (P[i]^2) = 1 \\ & for\ m = 0, ...2M-1\ and\ n = -\infty, ..., \infty \end{aligned} \tag{14}$$

where $\|.\|$ is the vector norm.

Using these normalizing factors, equation (13) can be further simplified as given below,

$$\begin{aligned} SLNR_{m,n} &= \frac{\|\psi_{m,n} Q_{m,n}\|^2}{2M\sigma^2 + \sum_{\substack{\acute{m}, m \neq \acute{m} \\ \acute{n}, n \neq \acute{n}}} \|\psi_{\acute{m},\acute{n}} Q_{m,n}\|^2} \\ &= \frac{\|\psi_{m,n} Q_{m,n}\|^2}{2M\sigma^2 + \|\tilde{\psi}_{m,n} Q_{m,n}\|^2} \end{aligned} \tag{15}$$

where,

$$\tilde{\psi}_{m,n} = \sum_{\substack{\acute{m},m\neq\acute{m}\\ \acute{n},n\neq\acute{n}}} \psi_{\acute{m},\acute{n}} \tag{16}$$

represents the sum of the channel coefficients except at sub-carrier (m) and time index (n).

Then if we can find a precoder matrix in such a way that the SLNR is maximized, the interference associated with the received signal can be eliminated. The optimization problem can be formulated as below.

$$\begin{aligned} Q_{m,n} \quad &= arg \max_{Q_{m,n}\epsilon C^{2m\times 1}} \frac{\|\psi_{m,n}Q_{m,n}\|^2}{2M\sigma^2 + \|\tilde{\psi}_{m,n}Q_{m,n}\|^2} \quad (17)\\ &subject\ to\ Tr(Q^*_{m,n}Q_{m,n}) = 1,\\ &for\ m = 0,...2M-1\ and\ n = -\infty,....,\infty. \end{aligned}$$

where Tr denotes the trace of a matrix.

Solving a similar kind of optimization problem is discussed in [9] in the context of multiuser beam-forming and hence we directly present closed form solution to the optimization problem in (17) as,

$$\begin{aligned} Q_{m,n} = \quad & max.eigenvector[(2M\sigma^2 + \tilde{\psi}^H_{m,n}\tilde{\psi}_{m,n})^{-1} \quad (18)\\ & \times\psi^H_{m,n}\psi_{m,n}] \end{aligned}$$

This implies that the optimum $Q_{m,n}$ is equal to the generalized eigenvector corresponding to the largest generalized eigenvalue of matrix pair $[{\psi_{m,n}}^H\psi_{m,n}, (2M\sigma^2+\tilde{\psi}^H_{m,n}\tilde{\psi}_{m,n})]$.

IV. Time Domain Equalization

For the comparison purpose, here we introduce two types of equalization methods, zero forcing (ZF) and minimum mean square error (MMSE), that can be applied at the receiver side in order to compensate the effect of multi-path channel fading and AWGN noise [15]. Then performance of these decoding algorithms are compared with the proposed scheme in (18).

In ZF Equalization method, the main idea is to find a set of filter coefficients in such a way that it removes the effect of channel from the received signal as, [15, eq.7.62].

$$\mathbf{e_{zf}} = (\mathbf{C}^\dagger\mathbf{C})^{-1}\mathbf{C}^\dagger \tag{19}$$

where e_{zf} is the equalizer matrix and $\mathbf{C}$ is the $1\times\acute{M}$channel matrix. "$(.)^\dagger$" denotes the Hermitian operation.

The MMSE equalization is based on finding equalizer coefficients so that it minimizes the variance of the difference between transmitted data and the signal at the equalizer output. The MMSE equalizer coefficients can be calculated as below [15, eq.7.81],

$$\mathbf{e_{mmse}} \quad = (\mathbf{C}^\dagger\mathbf{C} + \sigma_n^2 I)^{-1}\mathbf{C}^\dagger \tag{20}$$

where e_{mmse} is the equalizer matrix, σ_n^2 is the noise variance and *I* is the $\acute{M}\times\acute{M}$ identity matrix.

TABLE II

Simulation parameters

Parameter	FBMC	OFDM
Subcarriers ($\acute{M} = 2M$)	64	64
Number of used subcarriers	64	52
Symbols per Carrier (N)	2^{14}	2^{14}
Cyclic Prefix	-	16
Modulation	OQAM	QAM
Prototype filter Length (L)	64	64
Overlapping Factor (K)	4	1

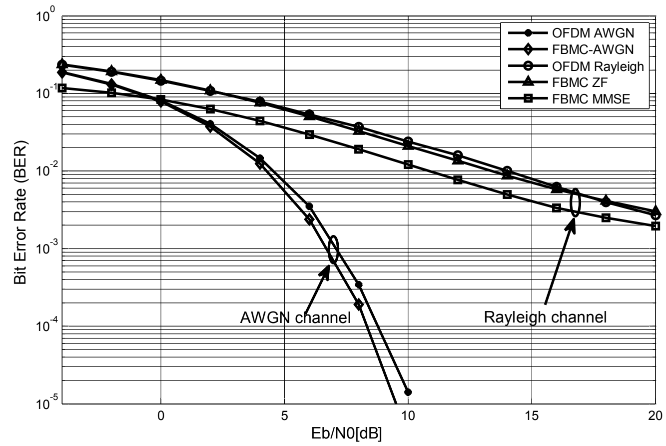

Fig. 3. BER vs SNR Performance of FBMC vs OFDM with different equalizers.

V. Results and Discussion

In this section we present Monte Carlo simulation results to demonstrate the effect on the system performance with respect to proposed precorder design and equalization techniques, described in [15]. Simulations have been carried out for both FBMC and OFDM with similar conditions and then system performance is investigated in terms of BER as a function of the SNR per bit at the receiver.

We assume perfect channel knowledge at both the transmitter and the receiver. The channel fading statistics follows Rayleigh fading and is assumed to be static at least for the duration of the symbol. The simulation parameters for both FBMC and OFDM are summarized in Table II.

The simulated BER performance for proposed FFT implementation of FBMC and OFDM systems are shown in Fig. 3. For comparison purposes, results for the AWGN channel is also plotted without considering the multipath propagation. It can be observed that FBMC has a small gain over OFDM in AWGN channel. In this case the receiver complexity is reasonably the same as in OFDM since no equalization is required. In multi-path scenario, ZF and MMSE equalization methods are applied to compensate the fading affect by Rayleigh channel and maximum likelihood decoding is used to estimate the transmitted symbols. In this case, performance gain is relatively small in FBMC compared to the receiver complexity. This is mainly due to the intrinsic interference in FBMC which is described in Table. I of section III.

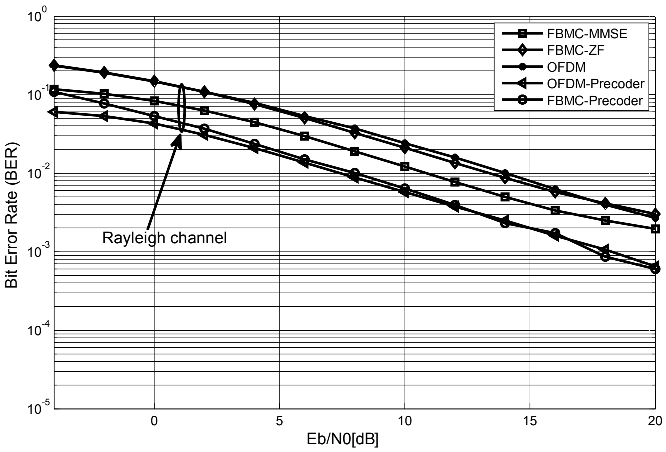

Fig. 4. BER vs SNR Performance of FBMC with Precoding.

In Fig. 4, we compare the FBMC with proposed precoder versus the FBMC with only equalization and with OFDM. It can be observed that proposed scheme outperforms both OFDM and FBMC with equalization. But, it shows almost equal performance with the precoded OFDM. However, FBMC comes with the added advantage of high spectral efficiency compared to OFDM as shown in Table III. This gain will be useful when combining FBMC with MIMO and cognitive radio in order to achieve higher bit rates than OFDM.

When we consider the computational complexity of the various precoding schemes described in section I, it can be observed that the proposed scheme is better since it does not need any additional FFT blocks or filters in order to calculate the precoding matrix. The comparison of FBMC and OFDM can be summarized as in Table III, where L_{eq} is the number of equalizer coefficients, β is referred to the roll-of factor of prototype filter, $\acute{M}_g$ is number of null-subcarriers and G is the length of the guard band of OFDM.

From Table III, we can observe that proposed FBMC has L_{eq} times lower complexity compared to FBMC scheme as described in [5] and higher spectral efficiency compared to CP-OFDM.

VI. Conclusion

In this paper, we have analyzed the effect of intrinsic interference in FBMC and then proposed a precoding scheme based on the signal-to-leakage-plus-noise ratio (SLNR) criterion in order to mitigate the effect of this interference as well as noise. We have obtained a closed form expression for the precoding matrix and compared its performance with general equalization methods. Furthermore, with the help of BER simulation graphs, it is shown that FBMC with proposed scheme gives better performance compared to conventional OFDM showing that FBMC is a good alternative for high speed wireless communication. In fact, the new scheme comes with the added advantage of minimizing receiver complexity significantly.

TABLE III
A comparison of FBMC and OFDM

Scheme	Complexity (IFFT Implementation)	Spectrum Efficiency
$FBMC_{Proposed}$	$2\acute{M}\log_2\acute{M}+\acute{M}$	$\frac{\acute{M}}{(\acute{M}+\beta)}$
$FBMC_{MMSE}$	$2\acute{M}\log_2\acute{M}+\acute{M}L_{eq}$	$\frac{\acute{M}}{(\acute{M}+\beta)}$
OFDM	$2\acute{M}\log_2\acute{M}$	$\frac{(\acute{M}-\acute{M}_g)}{(\acute{M}+G)}$

VII. Acknowledgment

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