Activation of Nomadic Relay Nodes in Dynamic Interference Environment for Energy Saving

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***Abstract*—This paper presents an optimization framework for energy savings in nomadic relay networks, where interference and load are changing over time due to varying assignments. We prove the existence of an explicit load function taking assignments as arguments and we show the function is continuously differentiable. Based on the properties of the load function, we design iterative relay and user association algorithm for energy savings, where the non-convex load constraints are linearly approximated such that each sub-problem is a linear program and hence can be solved efficiently. Simulation results confirm that our proposed algorithmic approach leads to a significant reduction of the energy consumption when compared with algorithms considering the worst-case interference.**

I. Introduction

An avalanche of mobile and wireless traffic volume is foreseen to increase a thousand-fold over the next decade. Meanwhile, a high potential of traffic dynamics and user mobility is envisaged in future 5G systems [1]. One of the main challenges is to satisfy these requirements in a cost-efficient manner, for which the relaying concept has proven to be a promising solution [2]. One promising 5G system component that tries to respond to the boosting traffic volume of the future information society is the concept of a nomadic network [3]. A nomadic network consists of randomly distributed non operator-deployed nodes (e.g., parked vehicles with on-board relay infrastructure and advanced backhaul antennas) offering the possibility to act as relay nodes (RNs) between user equipments (UEs) and base stations (BSs). While the location of operator-deployed relay nodes is optimized by means of network planning, the location of the RNs in a nomadic network, referred to as *nomadic RNs* or *nomadic nodes*, is out of control of a network operator, and therefore is considered to be random. Moreover, their availability and position may change in time (hence, the term "nomadic") due to battery state and node movement. The nomadic RNs operate in a self-organized fashion and are in general activated and deactivated based on capacity, coverage, load balancing or energy efficiency demands. Therefore, the concept of a nomadic network describes an effective extension of the cellular infrastructure that allows for a dynamic network deployment.

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As discussed in the previous work [4], the most important aspects in such networks include:

- Relay Selection (RS): selection and subsequent assignment of RNs to BSs;
- User Association (UA): assignment of UEs to RNs (two-hop relaying) or BSs (direct communications);
- Radio Resource Management (RRM): a flexible allocation of radio resources; note that here we assume that RNs are reusing the resources of BSs for the maximum spectral efficiency.

The optimization of UA has been intensively studied in the literature under constant interference scenarios [4][5][6]. The dynamic interference modal and load interference coupling function are further discussed in [7][8][9] and a UA algorithm is given in [10] for Load Balancing (LB). The topic of RS has been very well elaborated theoretically in [11], where the approach is based on link level metrics rather than a network-wide performance enhancement. The work in [12] and [13] focus on relay based load balancing network optimization algorithms, however, neither reuse among different network elements nor multi-cell deployments are not incorporated.

In this paper, we propose an optimization framework for the RS and UA problems in *nomadic relay-assisted multi-cell networks*, where flexible resource sharing among all the nodes is assumed. Based on our previous work in [4], we extend the optimization problem by taking into account the load dependent interference model. We show that under certain conditions, there exists an explicit function mapping the node assignments to the network loads. Moreover, we prove its differentiability, which is then used to design a practical optimization algorithm for energy saving.

The rest of the paper is organized as follows: Section II describes the system model with discussions on the load function. The optimization framework for minimizing the energy consumption is presented in Section III. Finally, simulation results are given in Section IV followed by some concluding remarks.

II. System Model and Problem Statement

In this paper, we consider the same network model as in [4], i.e., the downlink channel of a nomadic relay network with M BSs, K RNs and N UEs. The set of BSs, RNs and UEs are denoted by $\mathcal{B}$, $\mathcal{R}$ and $\mathcal{U}$, respectively. Furthermore, we use direct links, relay links and access links to denote the BS-UE, BS-RN and RN-UE links, respectively.[2] In this paper, we consider L3-RNs according to the Long Term Evolution (LTE) standard [14]. Such RNs are seen by the UEs as separate cells that have all the RRM functionalities of the BSs, e.g., the RNs are able to reuse the resources of the BS for the access link transmissions, while the relay links and direct links compete for resources allocated to the corresponding BS.

The amount of bandwidths (in Hz) at BSs and RNs are fixed and grouped in vectors $\mathbf{b}^{(\mathrm{m})} = (b_1^{(\mathrm{m})}, \ldots, b_M^{(\mathrm{m})})^T$ and $\mathbf{b}^{(\mathrm{k})} = (b_1^{(\mathrm{k})}, \ldots, b_K^{(\mathrm{k})})^T$, respectively. The vector of required minimum rates (in bit/s) of the UEs is denoted by $\mathbf{r}^{(\mathrm{n})} = (r_1^{(\mathrm{n})}, \ldots, r_N^{(\mathrm{n})})^T$. Note that each UE can be connected either to an RN or directly to a BS (not to both simultaneously), while an RN can only be connected to a BS. The Spectral Efficiency (SE) of a link (i,j) is assumed to be (in bit/s/Hz)

$$\omega_{i,j} = \zeta_b \cdot \log(1 + \zeta_s \cdot \tau_{i,j}) \tag{1}$$

where $0 \leq \zeta_b \leq 1$ is the bandwidth efficiency and $0 \leq \zeta_s \leq 1$ refers to the Signal to Interference plus Noise Ratio (SINR) efficiency, while $\tau_{i,j}$ denotes the corresponding SINR as defined in (2). Throughout the paper we take the following assumptions:

(A.1) The parameters $\mathbf{r}^{(\mathrm{n})}$, $\mathbf{b}^{(\mathrm{m})}$ and $\mathbf{b}^{(\mathrm{k})}$ are known parameters at a central network control unit or can be estimated reliably. We assume $\zeta_b = 1$ and $\zeta_s = 1$ for simplifying further analyses.

(A.2) While access links and direct links interfere with each other, the RNs use separate time-frequency resources on the relay links and access links. Hence the access links do not interfere with the relay links.

(A.3) In the previous work [4], we considered the worst-case interference model in order to ensure the bandwidth constraints. In order to further optimize the network, we adapt a more realistic interference model [7][9][10], where interference power is scaled by the load (denoted as ρ) of the interfering BSs.

As a result, the SINR of a link (i,j) is given by

$$\tau_{i,j} = \begin{cases} \frac{P_{i,j}}{\sum_{d \in \mathcal{B}, d \neq i} P_{d,j}\rho_d + \sigma_j} & \text{for } j \in \mathcal{R}, \\ \frac{P_{i,j}}{\sum_{d \in \mathcal{B} \bigcup \mathcal{R}, d \neq i} P_{d,j}\rho_d + \sigma_j} & \text{for } j \in \mathcal{U}, \end{cases} \tag{2}$$

where σ_j is the noise power at node j while ρ_d refers to the load of the node d which is defined as the ratio of the amount of used bandwidth for supporting the Quality of Service (QoS) of the assigned UEs to the available bandwidth at the node. We explain the load function ρ in the Section II-A in more detail. Note that $\tau_{i,j} > 0$ always holds, since both received power and interference plus noise are positive values. Furthermore, a positive $\tau_{i,j}$ exists also for the case when both $i,j \in \mathcal{R}$, however, we do not allow a connection between RNs in this work.

[2]Throughout this paper, notations with superscripts (m), (k) and (n) are variables associated with BSs, RNs and UEs, respectively, while notations with (m,n), (m,k) and (k,n) are referring to the direct links, relay links and access links, respectively.

A. Load Function

Let $x_{i,j}$ denote the assignment variable: $x_{i,j} = 1$ if there is an active connection between node i and node j, and $x_{i,j} = 0$ otherwise. We introduce the assignment matrix to be defined as

$$\mathbf{X} \triangleq \begin{pmatrix} \mathbf{X}^{(\mathrm{m,n})} & \mathbf{X}^{(\mathrm{m,k})} \\ \mathbf{X}^{(\mathrm{k,n})} & \mathbf{X}^{(\mathrm{k,k})} \end{pmatrix} \in \{0,1\}^{(M+K)\times(N+K)}, \tag{3}$$

where $\mathbf{X}^{(\mathrm{m,n})} \in \{0,1\}^{M\times N}$, $\mathbf{X}^{(\mathrm{k,n})} \in \{0,1\}^{K\times N}$ and $\mathbf{X}^{(\mathrm{m,k})} \in \{0,1\}^{M\times K}$ are assignment matrices for the direct, access and relay links, respectively.[3] Note that $\mathbf{X}^{(\mathrm{k,k})}$ implies the connection between RNs and in practice should be set to be an all zero matrix if multi-hop communication is not allowed. Further, we define $\bar{\mathbf{x}} \triangleq \mathrm{vec}(\mathbf{X}) := (\bar{x}_1, \ldots \bar{x}_{\mathrm{L}})^T$, where $\mathrm{L} = (M+K)\cdot(N+K)$ and $x_{i,j} = \bar{x}_{(M+K)\cdot(j-1)+i}$.

Throughout this paper, we relax the discrete condition on the assignment for continuous analysis. In this case, a node can be connected to multiple nodes [5] and a value of $x_{i,j}$ between $[0,1]$ indicates the portion of QoS that should be routed through link (i,j). To this end, we define $\boldsymbol{\rho} \triangleq [\begin{smallmatrix}\boldsymbol{\rho}_i^{(\mathrm{m})} \\ \boldsymbol{\rho}_i^{(\mathrm{k})}\end{smallmatrix}] = [\rho_1, \ldots \rho_M, \rho_{M+1}, \ldots, \rho_{M+K}]^T \in \mathbb{R}_+^{M+K}$, $i \in \mathcal{B} \bigcup \mathcal{R}$, to be the vector of loads at the BSs and RNs, where the i-th entry of the load vector can be calculated as

$$\begin{aligned} \rho_i &= \rho_i^{(1)} + \rho_i^{(2)}, \quad i \in \mathcal{B} \bigcup \mathcal{R} \\ &= \underbrace{\sum_{j \in \mathcal{U}} \frac{r_j^{(\mathrm{n})}}{b_i \omega_{i,j}(\boldsymbol{\rho})} x_{i,j}}_{\text{direct/access links}} + \underbrace{\sum_{k \in \mathcal{R}} \frac{r_k^{(\mathrm{k})}}{b_i \omega_{i,k}(\boldsymbol{\rho})} x_{i,k}}_{\text{relay links}} \\ &= \sum_{j \in \mathcal{U}} \frac{r_j^{(\mathrm{n})}}{b_i \omega_{i,j}(\boldsymbol{\rho})} x_{i,j} + \sum_{k \in \mathcal{R}} \sum_{j \in \mathcal{U}} \frac{r_j^{(\mathrm{n})}}{b_i \omega_{i,k}(\boldsymbol{\rho})} x_{k,j} x_{i,k}. \end{aligned} \tag{4}$$

Herein, $\rho_i^{(1)}$ and $\rho_i^{(2)}$ refer to the load corresponding to the UEs (direct/access links) and RNs (relay links), respectively, while $r_j^{(\mathrm{k})}$ is the rate requirement of an RN, which is the sum rate of all the UEs connected to this RN:

$$r_k^{(\mathrm{k})} = \sum_{j \in \mathcal{U}} r_j^{(\mathrm{n})} x_{k,j}, \quad \text{for } j \in \mathcal{U}, k \in \mathcal{R}. \tag{5}$$

In (4), the SE $\omega_{i,j}$ defined in (1) is written as a function of the load vector $\omega_{i,j}(\boldsymbol{\rho})$, since it is a function of the SINR which in turn depends on the load vector $\boldsymbol{\rho}$ according to (2). Therefore, the total load is determined by the function $\mathrm{F} : \mathbb{R}^{M+K+\mathrm{L}} \to \mathbb{R}^{M+K}$ given by

$$\boldsymbol{\rho} = \mathrm{F}(\boldsymbol{\rho}, \bar{\mathbf{x}}) = \mathrm{F}^{(1)}(\boldsymbol{\rho}, \bar{\mathbf{x}}) + \mathrm{F}^{(2)}(\boldsymbol{\rho}, \bar{\mathbf{x}}) \tag{6}$$

where $\rho_i^{(1)} = F_i^{(1)}(\boldsymbol{\rho}, \bar{\mathbf{x}})$ and $\rho_i^{(2)} = F_i^{(2)}(\boldsymbol{\rho}, \bar{\mathbf{x}})$.

The function is an extension of the load function in [7][9][10] to cellular networks with relay nodes. In particular, it is shown in [9] that any positive concave function on

[3]Throughout the paper, $\mathbf{1}^l(\mathbf{0}^l)$ refers to column vector of length l. If not specified, $\mathbf{1}(\mathbf{0})$ is a column vector with proper length for matrix operator. Furthermore, $\mathbf{1}^{m\times n}(\mathbf{0}^{m\times n})$ refers to an $m \times n$ matrix of ones(zeros).

$\mathbb{R}_+^{M+K}$ is a standard interference function that is defined by the following three axioms.

Definition 1 (Standard Interference Function (SIF) [15])**.** A function is an SIF if the following conditions hold,

- Positivity: $\forall_{\boldsymbol{\rho}\geq 0}\, f(\boldsymbol{\rho}) > 0$,
- Monotonicity: if $\boldsymbol{\rho} \geq \boldsymbol{\rho}' \geq 0$, then $f(\boldsymbol{\rho}) \geq f(\boldsymbol{\rho}')$,
- Scalability: $\forall_{\boldsymbol{\rho}\geq 0}\, \forall_{\alpha>1},\, \alpha f(\boldsymbol{\rho}) > f(\alpha\boldsymbol{\rho})$.

Now since the load function ρ_i is positive and concave on $\boldsymbol{\rho} \in \mathbb{R}_+^{M+K}$ for given assignments [9], we can conclude the following lemma

Lemma 1. For any given $\bar{\mathbf{x}}$, the load function $\mathrm{F}(\cdot, \bar{\mathbf{x}})$ defined by (6) is a standard interference function (SIF).

It is worth pointing out that the lemma can be also proven directly from Definition 1 by showing that the load function satisfies each of the conditions [10].

An immediate consequence of the lemma is the following proposition:

Proposition 1. Let $\mathcal{X} := \{\bar{\mathbf{x}} \in [0,1]^{\mathrm{L}} | \exists_{\rho>0} \boldsymbol{\rho} \geq \mathrm{F}(\boldsymbol{\rho}, \bar{\mathbf{x}})\}$ and assume that $\mathcal{X} \neq \emptyset$. Then, there exists a continuous function $\mathrm{G} : \mathcal{X} \mapsto \mathbb{R}^{M+K}$ relating $\boldsymbol{\rho}$ to $\bar{\mathbf{x}}$:

$$\boldsymbol{\rho} = \mathrm{G}(\bar{\mathbf{x}}),\ \bar{\mathbf{x}} \in \mathcal{X}\,. \tag{7}$$

Proof: Let $\bar{\mathbf{x}} \in \mathcal{X} \neq \emptyset$ be arbitrary. Then, by [15], we know that there exists $\boldsymbol{\rho}(\bar{\mathbf{x}}) > 0$ such that

$$\boldsymbol{\rho}(\bar{\mathbf{x}}) = \mathrm{F}(\boldsymbol{\rho}(\bar{\mathbf{x}}), \bar{\mathbf{x}}),\ \bar{\mathbf{x}} \in \mathcal{X}. \tag{8}$$

Moreover, $\boldsymbol{\rho}(\bar{\mathbf{x}}) > 0$ is the unique fixed-point of $\mathrm{F}(\cdot, \bar{\mathbf{x}})$. Now let

$$\mathrm{G}(\bar{\mathbf{x}}) = \boldsymbol{\rho}(\bar{\mathbf{x}}) = \mathrm{F}(\boldsymbol{\rho}(\bar{\mathbf{x}}), \bar{\mathbf{x}}) \tag{9}$$

and note that G maps elements of $\mathcal{X}$ into $\mathbb{R}_+^{M+K}$. Moreover, due to the uniqueness of the fixed point, we can conclude that, for any $\boldsymbol{\rho}(\bar{\mathbf{x}}^{(1)}) > 0$ and $\boldsymbol{\rho}(\bar{\mathbf{x}}^{(2)}) > 0$, $\boldsymbol{\rho}(\bar{\mathbf{x}}^{(1)}) \neq \boldsymbol{\rho}(\bar{\mathbf{x}}^{(2)})$ implies $\bar{\mathbf{x}}^{(1)} \neq \bar{\mathbf{x}}^{(2)}$. Therefore, $\mathrm{G} : \mathcal{X} \mapsto \mathbb{R}^{M+K}$ is a function. It is continuous because $\boldsymbol{\rho}(\bar{\mathbf{x}})$ and $\mathrm{F}(\cdot, \bar{\mathbf{x}})$ are both continuous, and the concatenation of continuous functions is continuous. ■

The unique fixed-point (if exists) in (8) can be found iteratively by the following fixed-point algorithm [15]:

$$\boldsymbol{\rho}(n+1) = \mathrm{F}(\boldsymbol{\rho}(n), \bar{\mathbf{x}}),\ \bar{\mathbf{x}} \in \mathcal{X}. \tag{10}$$

In other words, if $\mathcal{X} \neq \emptyset$, then the algorithm converges to the unique fixed-point $\boldsymbol{\rho}(\bar{\mathbf{x}})$ defined in (8).

Now our goal is to show that the load function in (8) is continuously differentiable on $\mathcal{X}$ (where the set $\mathcal{X}$ is defined in Proposition 1). To this end, we need the following definition:

Definition 2 (Generalized Diagonally Dominant Matrix (GDM) [16])**.** A matrix $\mathbf{H}$ is a *non-singular* GDM if

- $\mathbf{H}$ has non-negative elements on the diagonal and non-positive elements elsewhere;
- there exists an all positive vector $\mathbf{s}$, such that $\mathbf{H} \cdot \mathbf{s}$ is an all positive vector, i.e., $\mathbf{H} \cdot \mathbf{s} > 0$.

Proposition 2. $\mathrm{G}(\bar{\mathbf{x}})$ is *continuously differentiable* on $\mathcal{X} := \{\bar{\mathbf{x}} \in [0,1]^{\mathrm{L}} | \exists_{\rho>0} \boldsymbol{\rho} \geq \mathrm{F}(\boldsymbol{\rho}, \bar{\mathbf{x}})\}$.

Proof: By (9), it is sufficient to show that the function $\boldsymbol{\rho}(\bar{\mathbf{x}}), \bar{\mathbf{x}} \in \mathcal{X}$, is continuously differentiable. To this end, define $\tilde{\mathrm{F}} : \mathbb{R}^{M+K} \times \mathcal{X} \to \mathbb{R}^{M+K}$ to be $\tilde{\mathrm{F}}(\boldsymbol{\rho}, \bar{\mathbf{x}}) := \boldsymbol{\rho} - \mathrm{F}(\boldsymbol{\rho}, \bar{\mathbf{x}})$ and consider $\tilde{\mathrm{F}}(\boldsymbol{\rho}, \bar{\mathbf{x}}) = 0$, which is an implicit function since $\boldsymbol{\rho} = \boldsymbol{\rho}(\bar{\mathbf{x}})$ depends on $\bar{\mathbf{x}} \in [0,1]^L$. Therefore, by the implicit function theorem [17, Theorem 1], it suffices to show that for all $(\boldsymbol{\rho}, \bar{\mathbf{x}}) \in \mathbb{R}^{M+K} \times \mathcal{X}$: (i) $\tilde{\mathrm{F}}$ is differentiable, and (ii) the Jacobian $\mathbf{J}_{\tilde{\mathrm{F}}}^{\boldsymbol{\rho}}(\boldsymbol{\rho}, \bar{\mathbf{x}})$ with respect to $\boldsymbol{\rho}$ is invertible.[4] By looking at the analytical forms of the Jacobians in (12) and (22), it can be concluded that partial derivatives of $\tilde{\mathrm{F}}$ *exist and are continuous*. Therefore, $\tilde{\mathrm{F}}$ is differentiable according to [18] and (i) is satisfied. Moreover, (ii) follows from our analysis in the appendix, where it is shown that the Jacobian $\mathbf{J}_{\tilde{\mathrm{F}}}^{\boldsymbol{\rho}}(\boldsymbol{\rho}, \bar{\mathbf{x}})$ is a GDM on $\mathbb{R}^{M+K} \times \mathcal{X}$, and therefore $\mathbf{J}_{\tilde{\mathrm{F}}}^{\boldsymbol{\rho}}(\boldsymbol{\rho}, \bar{\mathbf{x}})$ is invertible on $\mathbb{R}^{M+K} \times \mathcal{X}$. This proves that $\boldsymbol{\rho}(\bar{\mathbf{x}})$ is continuously differentiable on $\bar{\mathbf{x}} \in \mathcal{X}$, which completes the proof. ■

Moreover, the Jacobian of G yields according to implicit function theorem

$$\mathbf{J}_{\mathrm{G}}^{\bar{\mathbf{x}}}(\bar{\mathbf{x}}) = -\mathbf{J}_{\tilde{\mathrm{F}}}^{\boldsymbol{\rho}}(\boldsymbol{\rho}, \bar{\mathbf{x}})^{-1} \mathbf{J}_{\tilde{\mathrm{F}}}^{\bar{\mathbf{x}}}(\boldsymbol{\rho}, \bar{\mathbf{x}}), \tag{11}$$

where $\mathbf{J}_{\tilde{\mathrm{F}}}^{\boldsymbol{\rho}}(\boldsymbol{\rho}, \bar{\mathbf{x}})$ can be found in the appendix and $\mathbf{J}_{\tilde{\mathrm{F}}}^{\bar{\mathbf{x}}}(\boldsymbol{\rho}, \bar{\mathbf{x}})$ is the Jacobian with respect to $\bar{\mathbf{x}}$ given by

$$\mathbf{J}_{\tilde{\mathrm{F}}_i}^{\bar{\mathbf{x}}_h} = \begin{cases} -\frac{r_j^{(\mathrm{n})}}{b_i \omega_{i,j}(\boldsymbol{\rho})} & \text{for } i = \tilde{i},\ j \in \mathcal{U}, \\ -\frac{r_k^{(\mathrm{k})}}{b_i \omega_{i,j}(\boldsymbol{\rho})} & \text{for } i = \tilde{i},\ j \in \mathcal{R}, \\ -\frac{r_j^{(\mathrm{n})}}{b_i \omega_{i,\tilde{i}}(\boldsymbol{\rho})} x_{i,\tilde{i}} & \text{for } i \in \mathcal{B},\ \tilde{i} \in \mathcal{R},\ j \in \mathcal{U}, \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

with $h = \tilde{i} + (j-1)(M+K)$ indicating the assignment between node $\tilde{i}$ and j.

III. Energy Saving Optimization

A. Problem Formulation

We can rewrite the energy saving optimization problem from our previous work [4] as[5]

$$\begin{aligned} \min_{\mathbf{X}} \quad & \mathrm{U}(\mathbf{X}) := \sum_{i=1}^{M+K} c_i ||\mathbf{e}_i^T \mathbf{X}\, \mathbf{1}||_0 && \text{(13a)} \\ \text{subject to} \quad & \mathbf{X}^T \cdot \mathbf{1} = \mathbf{1} && \text{(13b)} \\ & \boldsymbol{\rho} \leq \mathbf{1} && \text{(13c)} \\ & \mathbf{X} \in \{0,1\}^{(M+K)\times(N+K)}. && \text{(13d)} \end{aligned}$$

Herein, the vector $\mathbf{c} = (c_1, \ldots, c_{M+K}) \in \mathbf{R}_+^{(M+K)\times 1}$ denotes the energy consumption of active BSs and RNs,[6] while $\mathbf{e}_i^T$ is the i-th row of the identity matrix of the same size as $\mathbf{X}$, so that $\mathbf{e}_i^T \mathbf{X}$ yields the i-th row of $\mathbf{X}$. For any vector

[4]Throughout this paper, we denote $\mathbf{J}_{\mathrm{F}}^{\mathbf{x}}(\cdot)$ as the Jacobien of function $\mathrm{F}(\cdot)$ with respect to $\mathbf{x}$. Furthermore, the i-th row and the an entry i, j in $\mathbf{J}_{\mathrm{F}}^{\mathbf{x}}(\cdot)$ are denoted as $\mathbf{J}_{\mathrm{F}_i}^{\mathbf{x}}(\cdot)$ and $\mathbf{J}_{\mathrm{F}_i}^{\mathbf{x}_j}(\cdot)$, respectively

[5]For any two matrices $\mathbf{A}$ and $\mathbf{B}$ of the same size, $\mathbf{A} \circ \mathbf{B}$ denotes the Hadamard matrix product, while both $\mathbf{A} \leq \mathbf{B}$ and $\mathbf{A} \geq \mathbf{B}$ should be understood as element-wise comparisons.

[6]Following [5], we ignore the dynamic energy consumption at BS and RNs.

$\mathbf{x}$, its l_0-norm is denoted by $||\mathbf{x}||_0$, which is the cardinality of the set of non-zero vector elements of $\mathbf{x}$. Therefore, the objective (13a) reflects the energy consumption of the whole network infrastructure. Furthermore, (13b) and (13d) together states that a UE or an RN must be connected to the network. Finally, (13c) is the bandwidth constraint in [4] that can be expressed using matrices as follows

$$\boldsymbol{\rho} = \mathrm{F}(\boldsymbol{\rho}, \bar{\mathbf{x}}) = \mathbf{W}(\boldsymbol{\rho}, \mathbf{X}) \circ \mathbf{X} \leq \mathbf{1}, \tag{14}$$

where $\mathbf{W}(\boldsymbol{\rho}, \bar{\mathbf{x}}) = \begin{pmatrix} \mathbf{W}^{(\mathrm{m,n})} & \mathbf{W}^{(\mathrm{m,k})} \\ \mathbf{W}^{(\mathrm{k,n})} & \mathbf{0}^{K\times K} \end{pmatrix}$ with the non-zero blocks are defined as

$$\mathbf{W}^{(\mathrm{m,n})} = (\mathbf{1}^M [\mathbf{r}^{(\mathrm{n})}]^T) \circ \mathbf{\Omega}^{(\mathrm{m,n})}(\boldsymbol{\rho}), \tag{15a}$$
$$\mathbf{W}^{(\mathrm{k,n})} = (\mathbf{1}^K [\mathbf{r}^{(\mathrm{n})}]^T) \circ \mathbf{\Omega}^{(\mathrm{k,n})}(\boldsymbol{\rho}), \tag{15b}$$
$$\mathbf{W}^{(\mathrm{m,k})} = (\mathbf{1}^M [\mathbf{r}^{(\mathrm{k})}]^T) \circ \mathbf{\Omega}^{(\mathrm{m,k})}(\boldsymbol{\rho}). \tag{15c}$$

In (15), $\mathbf{\Omega}^{(\mathrm{m,n})}(\boldsymbol{\rho}) := (\frac{1}{\omega^{(\mathrm{m,n})}_{i,j}(\boldsymbol{\rho}) b^{(\mathrm{m})}_i})_{i,j} \in \mathbb{R}^{M\times N}$, $\mathbf{\Omega}^{(\mathrm{m,k})}(\boldsymbol{\rho}) := (\frac{1}{\omega^{(\mathrm{m,k})}_{i,j}(\boldsymbol{\rho}) b^{(\mathrm{k})}_i})_{i,j} \in \mathbb{R}^{M\times K}$ and $\mathbf{\Omega}^{(\mathrm{k,n})}(\boldsymbol{\rho}) := (\frac{1}{\omega^{(\mathrm{k,n})}_{i,j}(\boldsymbol{\rho}) b^{(\mathrm{m})}_i})_{i,j} \in \mathbb{R}^{K\times N}$. Moreover, $\mathbf{r}^{(\mathrm{k})}$ is the vector of RN rate requirements:

$$\mathbf{r}^{(\mathrm{k})} = \mathbf{X}^{(\mathrm{k,n})} \cdot \mathbf{r}^{(\mathrm{n})} = (r^{(\mathrm{k})}_1, \ldots, r^{(\mathrm{k})}_K)^T. \tag{16}$$

The problem in (13) is intractable due to the non-continuous nature of the objective function which requires certain relaxation. Furthermore, the implicit load function $\mathrm{F}(\boldsymbol{\rho}, \bar{\mathbf{x}})$ depends on both assignment $\bar{\mathbf{x}}$ and the load $\boldsymbol{\rho}$, resulting in high complexity for solving both $\bar{\mathbf{x}}$ and $\boldsymbol{\rho}$ at the same time. By assuming the worst-case interference in [4], we in fact removed the dependency of the load constraints on $\boldsymbol{\rho}$ by using an upper bound on $\mathrm{F}(\boldsymbol{\rho}, \bar{\mathbf{x}})$. In fact, by Lemma 1 and the monotonicity condition of Definition 1, we have for any $\bar{\mathbf{x}}$,

$$\mathrm{F}(\boldsymbol{\rho}, \bar{\mathbf{x}}) \leq \mathrm{F}(\mathbf{1}, \bar{\mathbf{x}}) = \mathbf{W}(\mathbf{1}, \mathbf{X}) \circ \mathbf{X}. \tag{17}$$

This constraint, however, can be shown to be non-convex [4], resulting in a higher level of complexity in solving the optimization problem.

B. Iterative Algorithm

In this paper, we apply relaxation and approximation techniques to the objective and constraints of the problem, such that an iterative process can be formulated where each iteration can be described as an LP [5].

For the objective function, we adopt the same approach as in [4] [5] by first approximating the objective using a concave function:[7]

$$\tilde{\mathrm{U}}_\epsilon(\mathbf{X}) := \sum_{i=1}^{M+K} \frac{c_i(\log(\epsilon + \mathbf{e}_i^T \mathbf{X}\mathbf{1}) - \log(\epsilon))}{\log(1+\epsilon^{-1})} \tag{18}$$

for some sufficiently small $\epsilon > 0$. Then, the function in (18) can be written in a more compact form that reveals the possibility of minimizing (18) by solving a sequence of linear problems:

$$\mathbf{X}^{(l)} \in \mathrm{argmin}_{\mathbf{X}} \sum_{i=1}^{M+K} \frac{c_i \mathbf{e}_i^T \mathbf{X}\mathbf{1}}{\mathbf{e}_i^T \mathbf{X}^{(l-1)}\mathbf{1} + \epsilon}. \tag{19}$$

Regarding the non-convex load constraints, we use a linear approximation of the load $\boldsymbol{\rho}$:

$$\boldsymbol{\rho} = \mathrm{G}(\bar{\mathbf{x}}) \approx \tilde{\mathrm{G}}(\bar{\mathbf{x}}) = \mathrm{G}(\bar{\mathbf{x}}^{(l-1)}) + \mathbf{J}^{\bar{\mathbf{x}}}_G(\bar{\mathbf{x}}^{(l-1)})(\bar{\mathbf{x}} - \bar{\mathbf{x}}^{(l-1)}), \tag{20}$$

where $\bar{\mathbf{x}}^{(l-1)}$ is the assignment from the last iteration and the function $\tilde{\mathrm{G}}$ exists and is continuous by Proposition 1 and Proposition 2. By assuming that the fixed-point iteration in (10) with the approximation in (20) is feasible and by relaxing the discrete condition (13d), the l-th iteration of the optimization problem is an LP:

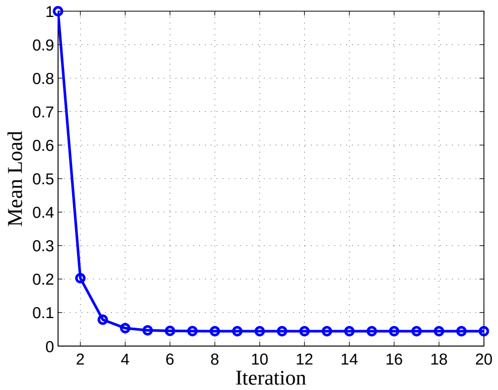
$$\min_{\mathbf{X}} \quad \tilde{\mathbf{U}}^{(l)}(\mathbf{X}) := \sum_{i=1}^{M+K} \frac{c_i \mathbf{e}_i^T \mathbf{X}\mathbf{1}}{\mathbf{e}_i^T \mathbf{X}^{(l-1)}\mathbf{1} + \epsilon} \tag{21a}$$
$$\text{subject to} \quad \mathbf{X}^T \cdot \mathbf{1} = \mathbf{1} \tag{21b}$$
$$\tilde{\mathrm{G}}(\bar{\mathbf{x}}) \leq \mathbf{1} \tag{21c}$$
$$\mathbf{X} \in [0,1]^{(M+K)\times(N+K)}. \tag{21d}$$

With all these in hand, we can formulate an iterative algorithm, denoted as Sequential Linear Reformulation (SLR), for solving the energy optimization problem in a dynamic interference environment as presented in Algorithm 1. The algorithm also applies to the femtocell network by neglecting the resource consumption on the relay link (i.e., $\mathrm{F}^{(2)} = 0$). Note that the linear approximation in (20) can not guarantee the feasibility of the original constraints in (14). Furthermore, if multipoint transmission is not possible, we need to heuristically map $\bar{\mathbf{x}}$ back to $\{0,1\}^{\mathrm{L}}$, by e.g., simple rounding operations. For these reasons, no proof for convergence can be given, however, simulations show that the algorithm has always converged.

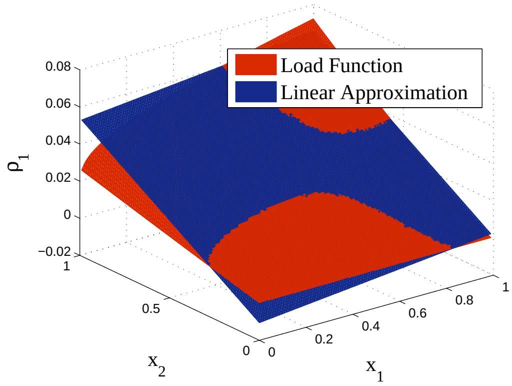
Algorithm 1: Sequential Linear Reformulation (SLR)

Let $l = 0$
Initialize $\mathbf{X}$ with $\mathbf{X}^l$ (and $\bar{\mathbf{x}}^l$)
Calculate the $\boldsymbol{\rho}^{(l)}$ using fixed-point iteration.
Choose a small value $\delta > 0$ as a termination scalar.
loop
  Solve the Linear Program (LP) (21):
  $\bar{\mathbf{x}}^{l+1} = \mathrm{argmin}_X \tilde{\mathbf{U}}^{(l+1)}(\bar{\mathbf{x}})$
  $\mathrm{G}(\bar{\mathbf{x}}^l)$ yields by fixed-point iteration (10)
  $\mathbf{J}^{\bar{\mathbf{x}}}_{\mathrm{G}}(\bar{\mathbf{x}}^l)$ can be calculated by (11)
  if $\|\tilde{\mathrm{U}}_\epsilon(\bar{\mathbf{x}}^{l+1}) - \tilde{\mathrm{U}}_\epsilon(\bar{\mathbf{x}}^l)\|_2 < \delta$ **then**
    break
  else
    $l \leftarrow l + 1$
  end if
end loop

[7] Note that $\lim_{\epsilon\to 0} \tilde{\mathrm{U}}_\epsilon(\mathbf{X}) = \mathrm{U}(\mathbf{X})$ and we interchangeably use $\bar{\mathbf{x}}$ and $\mathbf{X}$ as input argument for function $\mathrm{U}(\cdot)$, $\tilde{\mathrm{U}}_\epsilon(\cdot)$ and $\tilde{\mathbf{U}}^{(l)}(\cdot)$.



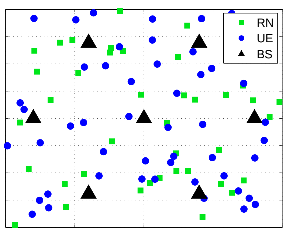
(a)



(b)

Fig. 1. (a) Convergence of the fix-point algorithm for the explicit load function, where x_1 = 0.7 and x_2 = 0.7 and (b) Load function and approaximation.

TABLE I
SIMULATION CONFIGURATIONS.

An example of the deployment scenario	
Deployment Scenario • 7 BSs in hexagon shape • Inter Site Distance (ISD): 1000 m • area length: 2.5 km • number of UEs: 50 • number of RNs: 20 • UE rate requirement: {0.01, 0.1, 1, 10} Mbps	
Transmission Parameters	
transmission power	46 dBm for BS & 23 dBm for RN
energy consumption	1000 Watt for BS & 50 Watt for RN
available bandwidth	20 MHz for BS & 20 MHz for RN
antenna configuration	2 antennas for BSs, RNs and UEs
Channel and Noise Parameters in [dB]	
path loss model for all links	as in Table A.2.1.1.2-3 in [19]
noise figure	5 dB at UE & RN
Simulation Parameters	
termination threshold δ	0.00001
l_0-norm approximation ϵ	0.0001

IV. Simulation Results

A. Load Function and Approaximation

We first illustrate the performance of the fixed-point algorithm and load approximation by a simple network with a UE with a constant QoS of 1 Mbps and two BSs with 10 MHz bandwidth. We denote ρ_1 and ρ_2 as the load of the first and the second BS, respectively, whereas x_1 and x_2 refer to the assignment of the UE to the corresponding BS. Starting with full load [1,1], Fig. 1 (a) shows that the load converges to the fixed-point within 10 iterations. Furthermore, it can be seen in Fig. 1 (b) that the linear approximation results in a reasonable estimation of the load function especially near the point of expansion.

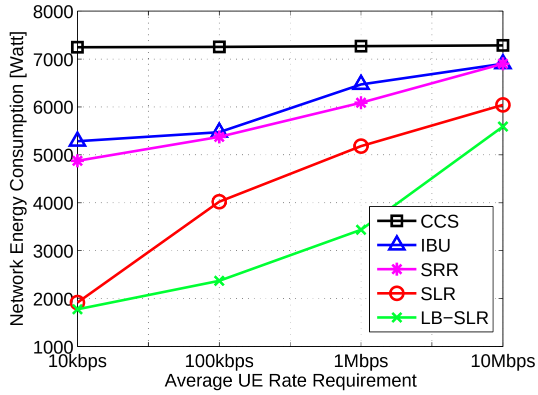


Fig. 2. Network energy consumption varying with UE rate requirements (Deployment: 7 BS, 20 RNs and 50 UE).

B. Energy Optimization Results

In the following, we evaluate the performance of a network including 7 LTE BSs with ISD of 1000 m. To identify the key factors that significantly influence the performance of our proposed algorithm, we use varying rate requirements for UEs, which reflects different data requirement profiles, e.g., higher data requirement at day time and lower data requirement at night. In order to generate sufficient statistics, 200 simulation runs are carried out. In each run, the nomadic RNs are randomly distributed within the area covered by the BS. An example of the deployment scenario and the simulation parameters are summarized in Tab. I.

Fig. 2 shows the network energy consumption using the following optimization approaches:

- Closest Cell Selection (CCS): UEs and RNs are connected to the closest access point,
- Iterative Backhaul Updating (IBU) as proposed in [4],
- Semi-Definite Programming (SDP) and Reformulation-Linearization Technique (RLT) Relaxation (SRR) as proposed in [4],
- SLR as proposed in Algorithm 1,
- LB-SLR: the RNs have wired backhauls, i.e., no resource consumption on the wireless backhaul, which is a lower bound for the SLR.

The first three reference algorithms have been also shown in our previous work [4], where a worst-case interference scenario is assumed. By considering dynamic interference, significant energy savings can be achieved (see SLR and LB-SLR). If the UE data requirement is very low (e.g., 10kbps) only two BSs in average need to be activated, which equals to a 40% of energy savings compared with the algorithms under worse-case assumption. Furthermore, wired backhaul (LB-SLR) does not improve the energy saving performance of nomadic network in a low rate scenario, since the loads generated by the wireless backhaul links are very small and can be easily satisfied. However, as the network load increases, the performance gains achieved by SLR are decreasing. Theoretically, if the network load is very huge, the performance gains in dynamic interference scenario will vanish and converge to the worse-case scenario.

V. Conclusion

In this paper, we presented a framework for relay selection and user association in multi-cell nomadic relay networks considering load dependent dynamic interference. With the help of SIF and GDM, we proof that there exists a continuously differentiable explicit load function that takes only the assignments as input variables. We approximate the non-convex load function by linear expansion, such that an iterative approach with LP as sub problems can be formulated for solving the energy saving problem. Simulation results show that the proposed algorithm significantly reduces the energy consumption compared with algorithms assuming worst-case interference.

Appendix

By definition of $\tilde{\mathrm{F}} : \mathbb{R}^{M+K} \times \mathcal{X} \to \mathbb{R}^{M+K}$, we have the Jacobian matrix $\mathbf{J}^{\rho}_{\tilde{\mathrm{F}}} = \mathbf{I} - \mathbf{J}^{\rho}_{\mathrm{F}}$ with element $\mathbf{J}^{\rho_{\tilde{i}}}_{\tilde{\mathrm{F}}_i}$ given by

$$\mathbf{J}^{\rho_{\tilde{i}}}_{\tilde{\mathrm{F}}_i} = \begin{cases} -\sum\limits_{j\in\mathcal{U}\bigcup\mathcal{R}} \frac{r_j^{(\mathrm{n})} x_{i,j}}{b_i\omega_{i,j}(\boldsymbol{\rho})} \frac{P_{\tilde{i},j}/P_{i,j}}{\ln(1+\tau_{i,j})(\tau_{i,j}^{-2}+\tau_{i,j}^{-1})} & i \neq \tilde{i}, \\ 1 & i = \tilde{i}. \end{cases} \tag{22}$$

Note that both i and $\tilde{i}$ are indices for transmitters (BS or RN) and j is the index for a receiver (UE or RN), therefore the notations $P_{i,j}$, $P_{\tilde{i},j}$ and $\tau_{i,j}$ above refer to the received power, interfering power from node $\tilde{i}$ and SINR for link (i,j), respectively. As a result, for any $\boldsymbol{\rho} > \mathbf{0}$,

$$\mathbf{J}^{\rho}_{\tilde{\mathrm{F}}_i}\boldsymbol{\rho} = \boldsymbol{\rho}_i - \sum_{\tilde{i}\neq i} \boldsymbol{\rho}_{\tilde{i}} \mathbf{J}^{\rho_{\tilde{i}}}_{\mathrm{F}_i}, \tag{23}$$

where $\mathbf{J}^{\rho}_{\tilde{\mathrm{F}}_i}$ is the i-th row of $\mathbf{J}^{\rho}_{\tilde{\mathrm{F}}}$ and

$$\sum_{\tilde{i}\neq i} \boldsymbol{\rho}_{\tilde{i}} \mathbf{J}^{\rho_{\tilde{i}}}_{\mathrm{F}_i} = \sum_{j\in\mathcal{U}\bigcup\mathcal{R}} \frac{r_j^{(\mathrm{n})} x_{i,j}}{b_i\omega_{i,j}(\boldsymbol{\rho})} \frac{(\sum_{\tilde{i}\neq i} \rho_{\tilde{i}} P_{\tilde{i},j})/P_{i,j}}{\ln(1+\tau_{i,j})(\tau_{i,j}^{-2}+\tau_{i,j}^{-1})}.$$

Now let $f(x) = (1+x)\ln(1+x) - x, x > 0$, and note that $f'(x) = \ln(1+x) > 0$. Hence, $f(x) > f(0) = 0$ implying that $(1+x)\ln(1+x)/x > 1$ for all $x > 0$. Thus,

$$\begin{aligned} & \ln(1+\tau_{i,j})(\tau_{i,j}^{-2}+\tau_{i,j}^{-1}) \\ = \; & {\tau_{i,j}}^{-1}\ln(1+\tau_{i,j})(1+\tau_{i,j})/\tau_{i,j} > {\tau_{i,j}}^{-1}. \end{aligned}$$

Furthermore, it can be easily verified that

$$(\textstyle\sum_{\tilde{i}\neq i} \rho_{\tilde{i}} P_{\tilde{i},j})/P_{i,j} < (\sum_{\tilde{i}\neq i} \rho_{\tilde{i}} P_{\tilde{i},j} + \sigma_j)/P_{i,j} = {\tau_{i,j}}^{-1}.$$

All these bounds together with (23) yield

$$\mathbf{J}^{\rho}_{\tilde{\mathrm{F}}_i}\boldsymbol{\rho} > \boldsymbol{\rho}_i - \sum_{j\in\mathcal{U}\bigcup\mathcal{R}} \frac{r_j^{(\mathrm{n})} x_{i,j}}{b_i\omega_{i,j}(\boldsymbol{\rho})} = 0. \tag{24}$$

Now since all the diagonal elements of $\mathbf{J}^{\rho}_{\tilde{\mathrm{F}}}$ are equal to one, while the off-diagonals are all negative, we conclude from Definition 2 and (24) that $\mathbf{J}^{\rho}_{\tilde{\mathrm{F}}}$ is an invertible GDM [16].

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