# Energy-Efficient Uplink Power Allocation in Multi-Cell MU-Massive-MIMO Systems

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*Abstract*—**In this paper, we investigate a joint uplink pilot and data power allocation strategy for energy-efficient communications under pilot contamination in multi-cell multi-user massive multiple-input multiple-output (MU-Massive-MIMO) systems. Assuming the maximum-ratio combining detection and a large number of antennas at the base station, the system energy efficiency (EE) is maximized over the individual power assignment between pilot and data transmissions of each user, subject to the per-user signal to interference-plus-noise ratio and per-user power constraints. To decentralize the optimization, game theory has to be applied, in which each cell competes against neighboring cells to maximize its own EE via fractional programming. The existence and uniqueness of Nash equilibrium of the game are discussed. Numerical results of the proposed algorithm are compared with the uplink power control scheme, which illustrates the significant advantage in EE by applying the per-user power allocation in multi-cell MU-Massive-MIMO.**

## I. Introduction

Multi-user massive multiple-input multiple-output (MU-Massive-MIMO) system with hundreds of antennas deployed at the base station (BS), which operates in time-division duplexing (TDD) mode, can serve a multiplicity of single-antenna users at the same time-frequency resource, while providing high throughput for each user and increased system energy efficiency (EE) [1] [2]. In particular, under the favorable propagation [3], any randomly selected channel vectors of different users tend to become pairwisely orthogonal when the number of BS antennas grows large. Therefore, in the single-cell scenario, any simple linear receiver (e.g. maximum-ratio combining (MRC)) is able to completely eliminate the uncorrelated intra-cell interference and noise with unlimited number of BS antennas [1]-[3].

However, when a multi-cell setup is considered, the channel state information (CSI) at the BS, which is estimated through uplink training, is contaminated due to the inevitable reuse of pilot sequences among neighboring cells [4]. This presents a fundamental limitation of multi-cell MU-Massive-MIMO system in practice, and deteriorates the throughput of linear detection/precoding in both uplink and downlink communications. Nevertheless, a considerable uplink EE is still achievable, as the data transmit power of each user can be reduced inversely proportional to the square-root of the number of BS antennas while maintaining the same throughput [2].

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In massive MIMO literature, e.g. [1]-[3], equal pilot and data power assignment for all users has been always assumed, which does not enable any power allocation scheme to improve the system EE. Recently, the work in [5] [6] has considered optimizing the downlink EE in single-cell setup: the focus of [5] is on the resource allocation with massive BS antennas for OFDMA; the massive MIMO design in [6] has assumed perfect CSI; however in both results, the pilot power allocation is not involved. While for massive MIMO in multi-cell scenario, the algorithm in [7] has employed an uplink pilot and data power control to minimize the total power consumption of all users, which also leads to an increased EE. However, to achieve the optimal uplink system EE by jointly allocating pilot/data powers, a *direct* approach in which the system EE is maximized as the target has to be specifically designed, which motivates the work in this paper.

Therefore, we propose in this paper a joint pilot and data power allocation scheme to improve the uplink system EE, subject to the per-user signal to interference-plus-noise ratio (SINR) and per-user power constraints. We take the pilot contamination into account and apply the low complexity MRC receive processing in order to avoid large matrix inversion. To keep the scheme as decentralized as possible, we refer to game theory, where each player (cell) is allowed to unilaterally optimize its own EE while keeping its rivals' strategies fixed. Moreover, as the received signal after MRC detection is coupled with interference – even if massive BS antennas are deployed, a maximum intra-cell interference temperature has to be inserted to parallelize the optimization target of each player [5]. This facilitates the design of efficient joint power allocation based on fractional programming [8]. The existence of the Nash equilibrium (NE) of the considered game is proved; while the uniqueness of NE is analytically indeterminate in general. The performance of joint power allocation is compared with the power control strategy [7] in simulations, where higher per-user SINR with a moderate sum power consumption is achieved by employing the proposed algorithm, thus demonstrates the attainable EE improvement in multi-cell MU-Massive-MIMO systems.

## II. System Model

We consider the uplink of multi-cell MU-Massive-MIMO system, where $L$ cells – each with $M$ antennas at the BS serving $K$ ($K \ll M$) users in the same time-frequency resource – operate in TDD mode. The matrix $\mathbf{G}_{li} = \mathbf{H}_{li}\mathbf{D}_{li}^{1/2}$ represents the flat-fading channel, and its $k$-th column, i.e. $\mathbf{g}_{lik} = \sqrt{\beta_{lik}}\mathbf{h}_{lik}$, is the propagation vector from the $k$-th user in cell $i$ to the $l$-th BS. In particular, the diagonal matrix $\mathbf{D}_{li} = \operatorname{diag}\{[\beta_{li1}, \ldots, \beta_{liK}]\} \in \mathbb{R}^{K\times K}$ describes the real-valued path-loss and large-scale fading which are assumed to be constant and known prior at the BSs, while the vector $\mathbf{h}_{lik} \in \mathbb{C}^{M\times 1}$ consists of independent and identically distributed (i.i.d.) complex Gaussian ($\mathcal{CN}(0,1)$) fast-fading coefficients [3]. Moreover, the length of the channel coherence interval for all cells is denoted as $T$ (in symbols), which in practice cannot afford orthogonal pilot sequences for all users in the system. As a compromise, only the users in the same cell transmit mutually orthogonal pilot sequences of length $\tau \geq K$, however, the same set of pilots is reused in the neighboring cells[1]. Denoting the pilot and data transmit power of all users in cell $i$ in diagonal matrices as $\mathbf{P}_{\mathrm{r},i} = \operatorname{diag}\{[p_{\mathrm{r},i1}, \ldots, p_{\mathrm{r},iK}]\}$ and $\mathbf{P}_{\mathrm{p},i} = \operatorname{diag}\{[p_{\mathrm{p},i1}, \ldots, p_{\mathrm{p},iK}]\}$, respectively, the received signal within a coherence interval at BS $l$ is given as

$$[\mathbf{Y}_{\mathrm{p},l} \quad \mathbf{Y}_{\mathrm{r},l}] = \sum_{i=1}^{L} \mathbf{G}_{li} \left[\sqrt{\tau}\mathbf{P}_{\mathrm{p},i}^{1/2}\mathbf{\Phi}^H \quad \mathbf{P}_{\mathrm{r},i}^{1/2}\mathbf{X}_i^H\right] + \mathbf{N}_l \quad (1)$$

where $\mathbf{Y}_{\mathrm{p},l} \in \mathbb{C}^{M\times\tau}$ and $\mathbf{Y}_{\mathrm{r},l} \in \mathbb{C}^{M\times(T-\tau)}$ are the received signal of pilot and data transmissions respectively, $\mathbf{\Phi} \in \mathbb{C}^{\tau\times K}$ denotes the (reused) pilot set from $K$ users such that $\mathbf{\Phi}^H\mathbf{\Phi} = \mathbf{I}_K$, $\mathbf{X}_i \in \mathbb{C}^{(T-\tau)\times K}$ contains data symbols from cell $i$, which are i.i.d. with zero mean and unit variance, and $\mathbf{N}_l \in \mathbb{C}^{M\times T}$ has i.i.d. $\mathcal{CN}(0,1)$ elements representing the uncorrelated normalized uplink noise.

### A. Channel Estimation and MRC Receive Processing

Relying on the received uplink training matrix $\mathbf{Y}_{\mathrm{p},l}$ in (1), the standard MMSE channel estimation is given by [9]

$$\hat{\mathbf{G}}_{li} = \mathbf{Y}_{\mathrm{p},l}\mathbf{\Phi}\left(\mathbf{I}_K + \tau\sum_{j=1}^{L}\mathbf{D}_{lj}\mathbf{P}_{\mathrm{p},j}\right)^{-1}\sqrt{\tau}\mathbf{D}_{li}\mathbf{P}_{\mathrm{p},i}^{1/2}. \quad (2)$$

It is straightforward to notice that except for the term $\mathbf{D}_{li}\mathbf{P}_{\mathrm{p},i}^{1/2}$ the rest part in (2) remains the same for all channel estimates $\hat{\mathbf{G}}_{li}, \forall i = 1, \ldots, L$, which reflects the impact of reusing the same set of pilot sequences among cells and results in the so-called pilot contamination problem [4].

Based on the channel estimate in (2), the MRC processed signal at BS $l$ for detecting the $t$-th ($\tau+1 \leq t \leq T$) transmitted data symbol from its $k$-th user is given by $r_{lk} = \hat{\mathbf{g}}_{llk}^H\mathbf{y}_{\mathrm{r},l}(t)$, where $\hat{\mathbf{g}}_{llk}$ and $\mathbf{y}_{\mathrm{r},l}(t)$ denote the column vectors of $\hat{\mathbf{G}}_{ll}$ and $\mathbf{Y}_{\mathrm{r},l}$, respectively. Similar to the manipulations in [2], the post-processing SINR $\theta_{lk}^{\mathtt{MRC}}$ is used to yield the uplink achievable ergodic rate, i.e. $R_{lk}^{\mathtt{MRC}} = \mathbb{E}\{\log_2(1+\theta_{lk}^{\mathtt{MRC}})\}$, which can be lower bounded by exploiting the convexity of $\log_2(1+1/x)$ and Jensen's inequality as

$$R_{lk}^{\mathtt{MRC}} \geq \tilde{R}_{lk}^{\mathtt{MRC}} \triangleq \log_2(1+\hat{\theta}_{lk}^{\mathtt{MRC}}) \quad (3)$$

where $\hat{\theta}_{lk}^{\mathtt{MRC}} \triangleq \left(\mathbb{E}\{1/\theta_{lk}^{\mathtt{MRC}}\}\right)^{-1}$, shown in (4) at the top of next page, can be also treated as the lower bound of the average SINR of user $k$ in cell $l$, since $\mathbb{E}\{1/\theta_{lk}^{\mathtt{MRC}}\} \geq 1/\mathbb{E}\{\theta_{lk}^{\mathtt{MRC}}\}$ [7].

[1] In principal, different cells can use different sets of pilot sequences, however this results in the effect of averaging the interference from all users in neighboring cells, which persists the pilot contamination [1].

### B. System Energy Efficiency

For the uplink transmission with MRC detection in a coherence interval $T$, the EE of user $k$ in cell $l$ is defined as the spectral efficiency $\frac{T-\tau}{T}\tilde{R}_{lk}^{\mathtt{MRC}}$ divided by the average transmit power per symbol $\frac{1}{T}(\tau p_{\mathrm{p},lk} + (T-\tau)p_{\mathrm{r},lk})$, i.e.

$$\eta_{lk}^{\mathtt{MRC}} \triangleq \frac{\frac{T-\tau}{T}\tilde{R}_{lk}^{\mathtt{MRC}}}{\frac{1}{T}(\tau p_{\mathrm{p},lk} + (T-\tau)p_{\mathrm{r},lk})} = \frac{\tilde{R}_{lk}^{\mathtt{MRC}}}{p_{\mathrm{r},lk} + \frac{\tau}{T-\tau}p_{\mathrm{p},lk}}. \quad (6)$$

Since the transmit powers of different users cannot be shared between each other and so are their data throughput and EE, we define the system EE of multi-cell MU-Massive-MIMO as the summation over individual EEs of all users, i.e.

$$\eta^{\mathtt{MRC}} \triangleq \sum_{l=1}^{L}\eta_l^{\mathtt{MRC}} = \sum_{l=1}^{L}\sum_{k=1}^{K}\eta_{lk}^{\mathtt{MRC}} \quad (7)$$

where $\eta_l^{\mathtt{MRC}} \triangleq \sum_{k=1}^{K}\eta_{lk}^{\mathtt{MRC}}$ is the sum EE in cell $l$ [2].

**Remark 1.** *Unlike the downlink BS power consumption model in [5] [6], we do not consider the power consumed by the transceiver chain at the BSs which may scale with $M$ and $K$, as our focus in this paper is the joint allocation of pilot and data transmit powers of all users in the uplink.*

### C. Optimization Problem Formulation

Similar to [2], we consider here the minimum length of pilot sequences, i.e. $\tau = K$. By defining $\mathbf{p}_{\mathrm{r}} \triangleq \left[\mathbf{p}_{\mathrm{r},1}^T, \ldots, \mathbf{p}_{\mathrm{r},L}^T\right]^T$ with $\mathbf{p}_{\mathrm{r},l} \triangleq [p_{\mathrm{r},l1}, \ldots, p_{\mathrm{r},lK}]^T$ and $\mathbf{p}_{\mathrm{p}} \triangleq \left[\mathbf{p}_{\mathrm{p},1}^T, \ldots, \mathbf{p}_{\mathrm{p},L}^T\right]^T$ with $\mathbf{p}_{\mathrm{p},l} \triangleq [p_{\mathrm{p},l1}, \ldots, p_{\mathrm{p},lK}]^T$, we introduce the optimization problem for energy-efficient uplink communication as

$$\begin{aligned} &\underset{(\mathbf{p}_{\mathrm{r}},\mathbf{p}_{\mathrm{p}})}{\text{maximize}} \quad \eta^{\mathtt{MRC}}(\mathbf{p}_{\mathrm{r}}, \mathbf{p}_{\mathrm{p}}) && (8)\\ &\text{subject to} \quad \begin{array}{l}\hat{\theta}_{lk}^{\mathtt{MRC}} \geq \gamma_{lk} \\ Kp_{\mathrm{p},lk} + (T-K)p_{\mathrm{r},lk} \leq P_{lk}\end{array} && \forall(l,k)\end{aligned}$$

where the system EE in (7) is maximized with respect to (w.r.t.) the joint selection of pilot and data powers, while fulfilling the per-user SINR $\gamma_{lk}$ and transmit power $P_{lk}$ constraints. Note that $P_{lk}$ may be interpreted as the energy budget per coherence interval if $T$ and $\tau$ are counted in time instead of symbols. For the sake of brevity, we do not explicitly list the power constraint $\mathbf{p}_{\mathrm{r}}, \mathbf{p}_{\mathrm{p}} > \mathbf{0}$, nevertheless it is in fact considered throughout the paper.

## III. Game Theory on EE Maximization

In this section, we reformulate the system EE maximization based on the non-cooperative game, in which each player's optimization problem is further modified in order to be solved by fractional programming.

### A. Reformulation of EE Maximization via Game Theory

To avoid the cooperation among cells which is required in solving the maximization problem in (8), we refer to a non-cooperative approach in game theory, where each cell acts as a player and tries to maximize its EE unilaterally, while assuming fixed resource allocation in neighboring cells, until a NE[2] of the game is achieved [10]. Hence we define a

[2] A NE of the non-cooperative EE maximization game is a set of power allocation that no cell can unilaterally improve its own EE by selecting a different set of pilot/data powers [10, Def. 1].

$$\hat{\theta}_{lk}^{\mathtt{MRC}} = \frac{(M-1)p_{\mathrm{r},lk}p_{\mathrm{p},lk}\beta_{llk}^2}{(M-1)\sum_{i\neq l}^{L} p_{\mathrm{r},ik}p_{\mathrm{p},ik}\beta_{lik}^2 + \left(\sum_{i=1}^{L}\sum_{\kappa=1}^{K} p_{\mathrm{r},i\kappa}\beta_{li\kappa}+1\right)\left(\sum_{i=1}^{L} p_{\mathrm{p},ik}\beta_{lik}+\frac{1}{\tau}\right) - \sum_{i=1}^{L} p_{\mathrm{r},ik}p_{\mathrm{p},ik}\beta_{lik}^2} \tag{4}$$

$$\triangleq \frac{(M-1)p_{\mathrm{r},lk}p_{\mathrm{p},lk}\beta_{llk}^2}{(M-2)I_{\mathrm{rp},-l} + \left(\sum_{\kappa\neq k}^{K} p_{\mathrm{r},l\kappa}\beta_{ll\kappa} + I_{\mathrm{r},-l}\right)\left(p_{\mathrm{p},lk}\beta_{llk}+I_{\mathrm{p},-l}\right) + p_{\mathrm{r},lk}\beta_{llk}I_{\mathrm{p},-l}} \tag{5}$$

game $\mathcal{G}^{\mathtt{MRC}}$ consisting of $L$ players, in which the optimization problem of the $l$-th player (cell) is to determine, for given feasible $(\mathbf{p}_{\mathrm{r},i},\mathbf{p}_{\mathrm{p},i})_{i\neq l}^{L}$, a solution $(\mathbf{p}_{\mathrm{r},l},\mathbf{p}_{\mathrm{p},l})$ in order to

$$\begin{aligned} \underset{(\mathbf{p}_{\mathrm{r},l},\mathbf{p}_{\mathrm{p},l})}{\text{maximize}} \quad & \eta_l^{\mathtt{MRC}}(\mathbf{p}_{\mathrm{r},l},\mathbf{p}_{\mathrm{p},l}) \\ \text{subject to} \quad & \begin{matrix} \hat{\theta}_{lk}^{\mathtt{MRC}} \geq \gamma_{lk} \\ Kp_{\mathrm{p},lk} + (T-K)p_{\mathrm{r},lk} \leq P_{lk} \end{matrix} \quad \forall k \end{aligned} \tag{9}$$

where $\eta_l^{\mathtt{MRC}}(\mathbf{p}_{\mathrm{r},l},\mathbf{p}_{\mathrm{p},l})$ depends only on the power allocation of users in cell $l$.

Accordingly, we can rewrite the denominator of $\hat{\theta}_{lk}^{\mathtt{MRC}}$, as shown in (5), where $I_{\mathrm{r},-l} \triangleq \sum_{i\neq l}^{L}\sum_{\kappa=1}^{K} p_{\mathrm{r},i\kappa}\beta_{li\kappa} + 1$, $I_{\mathrm{p},-l} \triangleq \sum_{i\neq l}^{L} p_{\mathrm{p},ik}\beta_{lik} + \frac{1}{K}$ and $I_{\mathrm{rp},-l} \triangleq \sum_{i\neq l}^{L} p_{\mathrm{r},ik}p_{\mathrm{p},ik}\beta_{lik}^2$ comprise solely the inter-cell contribution and can be hence treated as constant in the $l$-th player's optimization problem. However, the remained term $\sum_{\kappa\neq k}^{K} p_{\mathrm{r},l\kappa}\beta_{ll\kappa}$, which consists of the interference from intra-cell users, correlates the $k$-th $\hat{\theta}_{lk}^{\mathtt{MRC}}$ (so as $\tilde{R}_{lk}^{\mathtt{MRC}}$ and $\eta_{lk}^{\mathtt{MRC}}$) with all other users in the same cell. Therefore, the optimization target $\eta_l^{\mathtt{MRC}}$ in (9), which is the sum of multiple fractional terms, *cannot* be parallelized, i.e. $\max\sum_{k=1}^{K}\eta_{lk}^{\mathtt{MRC}} \neq \sum_{k=1}^{K}\max\eta_{lk}^{\mathtt{MRC}}$. This makes the problem in general very difficult to solve.

### *B. Modified Game of EE Maximization*

To this end, an additional constraint has to be introduced to the $l$-th player's problem in (9), i.e.

$$\sum_{\kappa\neq k}^{K} p_{\mathrm{r},l\kappa}\beta_{ll\kappa} \leq I, \quad \forall k \tag{10}$$

which acts as the maximum intra-cell interference temperature [5]. Furthermore, replacing $\sum_{\kappa\neq k}^{K} p_{\mathrm{r},l\kappa}\beta_{ll\kappa}$ with $I$ in (5) yields

$$\hat{\theta}_{\mathrm{lb},lk}^{\mathtt{MRC}} \triangleq \frac{(M-1)p_{\mathrm{r},lk}p_{\mathrm{p},lk}\beta_{llk}^2}{p_{\mathrm{r},lk}\beta_{llk}I_{\mathrm{p},-l} + p_{\mathrm{p},lk}\beta_{llk}(I+I_{\mathrm{r},-l}) + \iota} \tag{11}$$

where $\iota \triangleq (M-2)I_{\mathrm{rp},-l} + I_{\mathrm{p},-l}(I+I_{\mathrm{r},-l})$ is irrelevant of the transmit power allocation of the user considered. It is easy to see that the updated feasible set $\mathcal{X}_l^{\mathtt{MRC}} = \left\{ (\mathbf{p}_{\mathrm{r},l},\mathbf{p}_{\mathrm{p},l}) \,\middle|\, \begin{matrix} \hat{\theta}_{\mathrm{lb},lk}^{\mathtt{MRC}}(p_{\mathrm{r},lk},p_{\mathrm{p},lk}) \geq \gamma_{lk} \\ Kp_{\mathrm{p},lk}+(T-K)p_{\mathrm{r},lk} \leq P_{lk} \;\; \forall k \\ \sum_{\kappa\neq k}^{K} p_{\mathrm{r},l\kappa}\beta_{ll\kappa} \leq I \end{matrix} \right\}$ has a reduced size, which results in a performance lower bound of the original problem. Then, the achievable uplink data rate of user $k$ in cell $l$ (cf. (3)) can be (further) lower bounded by $\tilde{R}_{lk}^{\mathtt{MRC}} > \tilde{R}_{\mathrm{lb},lk}^{\mathtt{MRC}} \triangleq \log_2\left(\hat{\theta}_{\mathrm{lb},lk}^{\mathtt{MRC}}\right)$, which leads to a modified maximization target for (9), i.e. $\hat{\eta}_l^{\mathtt{MRC}} = \sum_{k=1}^{K}\eta_{\mathrm{lb},lk}^{\mathtt{MRC}}$ with

$$\eta_{\mathrm{lb},lk}^{\mathtt{MRC}} = \frac{\tilde{R}_{\mathrm{lb},lk}^{\mathtt{MRC}}\left(p_{\mathrm{r},lk},p_{\mathrm{p},lk};I_{\mathrm{r},-l},I_{\mathrm{p},-l},I_{\mathrm{rp},-l},I\right)}{p_{\mathrm{r},lk}+\frac{K}{T-K}p_{\mathrm{p},lk}}. \tag{12}$$

It is worth to note that for given interference temperature $I$ the value of $\eta_{\mathrm{lb},lk}^{\mathtt{MRC}}$ in (12) is decoupled from intra-cell users (other than user $k$) in cell $l$. As a result, we can define a *modified* game $\mathcal{G}_I^{\mathtt{MRC}}$, in which the optimization task of the $l$-th player is to determine, with given $(\mathbf{p}_{\mathrm{r},i},\mathbf{p}_{\mathrm{p},i})_{i\neq l}^{L}$, a solution tuple $(\mathbf{p}_{\mathrm{r},l},\mathbf{p}_{\mathrm{p},l}) \in \mathcal{X}_l^{\mathtt{MRC}}$, such that $\max\hat{\eta}_l^{\mathtt{MRC}}$ or equivalently $\max\eta_{\mathrm{lb},lk}^{\mathtt{MRC}}$ for all $k$ is achieved.

**Remark 2.** *Although the modification from the original game $\mathcal{G}^{\mathtt{MRC}}$ to $\mathcal{G}_I^{\mathtt{MRC}}$ may yield a solution which is sub-optimal, it facilitates the design of an efficient power allocation based on fractional programming in Section III-C, whose result can be treated as the performance lower bound of the achievable uplink system EE.*

### *C. Fractional Programming in Game $\mathcal{G}_I^{\mathtt{MRC}}$*

Since the optimization task of each player in game $\mathcal{G}_I^{\mathtt{MRC}}$ is equivalent to finding $K$ optimal user EEs, which are all in the fractional form with convex denominator and concave numerator (cf. (12)), we can apply here an efficient parametric convex fractional programming [8], such that the optimal EE of user $k$ in cell $l$, i.e. $q_{lk}^* \triangleq \max\eta_{\mathrm{lb},lk}^{\mathtt{MRC}} = \frac{\tilde{R}_{\mathrm{lb},lk}^{\mathtt{MRC}}(p_{\mathrm{r},lk}^*,p_{\mathrm{p},lk}^*)}{p_{\mathrm{r},lk}^*+\frac{K}{T-K}p_{\mathrm{p},lk}^*}$, is attained, if and only if $\max\eta_{\mathrm{fp},lk}^{\mathtt{MRC}}(p_{\mathrm{r},lk},p_{\mathrm{p},lk};q_{lk}^*) \stackrel{!}{=} 0$, where the equivalent objective function $\eta_{\mathrm{fp},lk}^{\mathtt{MRC}}$ is given as

$$\begin{aligned} \eta_{\mathrm{fp},lk}^{\mathtt{MRC}} \triangleq & \log_2\left(\frac{(M-1)p_{\mathrm{r},lk}p_{\mathrm{p},lk}\beta_{llk}^2}{p_{\mathrm{r},lk}\beta_{llk}I_{\mathrm{p},-l}+p_{\mathrm{p},lk}\beta_{llk}(I+I_{\mathrm{r},-l})+\iota}\right) \\ & - q_{lk}^*\left(p_{\mathrm{r},lk}+\frac{K}{T-K}p_{\mathrm{p},lk}\right). \end{aligned} \tag{13}$$

The "equivalence" here means that both maximizations result in the same power allocation.

Moreover, as proved in [5], the function $F_{lk}^{\mathtt{MRC}}(q_{lk}) \triangleq \max\eta_{\mathrm{fp},lk}^{\mathtt{MRC}}(q_{lk})$ is strictly monotonic decreasing in $q_{lk}$, and the inequality $F_{lk}^{\mathtt{MRC}}(q_{lk}) \geq 0$ holds true $\forall q_{lk} \neq q_{lk}^*$. Therefore, the optimal (cell) EE of the $l$-th player in game $\mathcal{G}_I^{\mathtt{MRC}}$, i.e. $\max\sum_{k=1}^{K}\eta_{\mathrm{lb},lk}^{\mathtt{MRC}} = \sum_{k=1}^{K}q_{lk}^*$, is achieved, if and only if $\max\sum_{k=1}^{K}\eta_{\mathrm{fp},lk}^{\mathtt{MRC}} = \sum_{k=1}^{K}\max\eta_{\mathrm{fp},lk}^{\mathtt{MRC}}(p_{\mathrm{r},lk},p_{\mathrm{p},lk};q_{lk}^*) \stackrel{!}{=} 0$.

### *D. Sketch of EE Maximization Algorithm*

Previous discussions imply that the solution to the original system EE optimization in (8) can be approximated by executing $\mathcal{G}_I^{\mathtt{MRC}}$ in a two-loop algorithm as described below.

In the outer loop, game theory is applied, where each player (cell) sequentially maximizes its own EE – by repeating its inner loop with fixed strategies of other players – until a NE is achieved. In particular, only the user geometry information (i.e. $\beta_{lik}$) has to be shared among the BSs in this loop, which varies slowly in accordance with the coherence time.

While in the inner loop of each player, the cell EE is optimized by maximizing $K$ user EEs at the same time via fractional programming, i.e.

$$\begin{aligned} \underset{(p_{\mathrm{r},lk}(n),\, p_{\mathrm{p},lk}(n))}{\text{maximize}} \quad & \eta^{\mathtt{MRC}}_{\mathrm{fp},lk}\left(p_{\mathrm{r},lk}(n), p_{\mathrm{p},lk}(n); q_{lk}(n)\right) \quad \forall k \\ \text{subject to} \quad & (\mathbf{p}_{\mathrm{r},l}(n), \mathbf{p}_{\mathrm{p},l}(n)) \in \mathcal{X}^{\mathtt{MRC}}_{l} \end{aligned} \tag{14}$$

in which the optimal user EE $q^{*}_{lk}$ is approached by iteratively updating $q_{lk}(n+1) = \frac{\tilde{R}^{\mathtt{MRC}}_{\mathrm{lb},lk}(p_{\mathrm{r},lk}(n), p_{\mathrm{p},lk}(n))}{p_{\mathrm{r},lk}(n) + \frac{K}{T-K} p_{\mathrm{p},lk}(n)}$ with $n$ denoting the iteration index. This is usually referred to as the Dinkelbach method, and its fast convergence (in approximate 6 circles) is insured [5] [8].

### E. Existence and Uniqueness of NE in Game $\mathcal{G}^{\mathtt{MRC}}_{I}$

Given that the $l$-th player's optimization problem is feasible, the solution set $\mathcal{X}^{\mathtt{MRC}}_{l}$ is non-empty, convex and compact, thus the existence of a NE in the game only depends on the joint quasi-concavity of the objective function $\sum_{k=1}^{K} \eta^{\mathtt{MRC}}_{\mathrm{lb},lk}$ w.r.t. the optimization variables $\mathbf{p}_{\mathrm{r},l}$ and $\mathbf{p}_{\mathrm{p},l}$ [10, Thm. 1]. Equivalently, we can also evaluate the Hessian matrix of the fractional program $\sum_{k=1}^{K} \eta^{\mathtt{MRC}}_{\mathrm{fp},lk}$ for fixed $q^{*}_{lk}$ (cf. (13)), i.e.

$$H\left(\sum_{k=1}^{K} \eta^{\mathtt{MRC}}_{\mathrm{fp},lk}\right) = \operatorname{diag}\left\{\left[H\left(\eta^{\mathtt{MRC}}_{\mathrm{fp},l1}\right), \ldots, H\left(\eta^{\mathtt{MRC}}_{\mathrm{fp},lK}\right)\right]\right\} \tag{15}$$

where the $k$-th diagonal element $H\left(\eta^{\mathtt{MRC}}_{\mathrm{fp},lk}\right)$ is the $2\times 2$ Hessian matrix of $\eta^{\mathtt{MRC}}_{\mathrm{fp},lk}$ (w.r.t. $p_{\mathrm{r},lk}$ and $p_{\mathrm{p},lk}$) and can be shown to be negative definite (the proof is omitted here due to space limit). Hence it follows immediately that $H\left(\sum_{k=1}^{K} \eta^{\mathtt{MRC}}_{\mathrm{fp},lk}\right)$ in (15) is also negative definite, and the existence of a NE is guaranteed.

The uniqueness of the NE is analytically indeterminate [10, Thm. 2], since the monotonicity and scalability of the $k$-th response function of the $l$-th player's power allocation, which is defined as $F^{\mathtt{MRC}}_{\mathrm{R},lk}\left(I_{\mathrm{r},-l}, I_{\mathrm{p},-l}, I_{\mathrm{rp},-l}; q^{*}_{lk}, I\right) \triangleq \arg\max \eta^{\mathtt{MRC}}_{\mathrm{fp},lk}$, cannot be evaluated. This results from the fact that the optimization variables $(p_{\mathrm{r},lk}, p_{\mathrm{p},lk})$ appear in the denominator of the first term of $\eta^{\mathtt{MRC}}_{\mathrm{fp},lk}$ in (13). Other approaches to verify the uniqueness may exist, which is an interesting topic for future work. Nevertheless, simulations carried out in Section V empirically suggest a unique NE of game $\mathcal{G}^{\mathtt{MRC}}_{I}$.

## IV. Review of Uplink Power Control

The uplink power control problem, aiming at jointly minimizing the sum pilot and data power of all users subject to the per-user SINR and power constraints, is given by

$$\begin{aligned} \underset{\mathbf{p}_{\mathrm{r}}, \mathbf{p}_{\mathrm{p}}}{\text{minimize}} \quad & \sum_{l=1}^{L}\sum_{k=1}^{K} \left(K p_{\mathrm{p},lk} + (T-K) p_{\mathrm{r},lk}\right) \qquad (16) \\ \text{subject to} \quad & \begin{matrix} \hat{\theta}^{\mathtt{MRC}}_{lk} \geq \gamma_{lk} \\ K p_{\mathrm{p},lk} + (T-K) p_{\mathrm{r},lk} \leq P_{lk} \end{matrix} \quad \forall (l,k). \end{aligned}$$

To solve the above problem, the introduced algorithm in [7], based on the alternating optimization approach [11], consists of a main loop with two consecutive phases: in the pilot power control phase, the pilot power is minimized for given data power; while in the subsequent data power control phase, the data power is minimized with the obtained pilot power; and this process is repeated until the optimal pilot and data power allocation is achieved.

For MRC detection, the per-user SINR constraint in both phases can be formulated into vector inequalities, i.e. $\mathbf{p}_{\mathrm{p}} \geq \boldsymbol{I}_{\mathrm{p}}(\mathbf{p}_{\mathrm{p}})$ and $\mathbf{p}_{\mathrm{r}} \geq \boldsymbol{I}_{\mathrm{r}}(\mathbf{p}_{\mathrm{r}})$, where the interference functions $\boldsymbol{I}_{\mathrm{p}}(\mathbf{p}_{\mathrm{p}})$ and $\boldsymbol{I}_{\mathrm{r}}(\mathbf{p}_{\mathrm{r}})$ are standard [7]. Therefore, an iterative method, with $n_{\mathrm{p}}$ and $n_{\mathrm{r}}$ denoting the iteration indices in the corresponding phases, can be applied, i.e. $\mathbf{p}_{\mathrm{p}}(n_{\mathrm{p}}+1) = \boldsymbol{I}_{\mathrm{p}}(\mathbf{p}_{\mathrm{p}}(n_{\mathrm{p}}))$ and $\mathbf{p}_{\mathrm{r}}(n_{\mathrm{r}}+1) = \boldsymbol{I}_{\mathrm{r}}(\mathbf{p}_{\mathrm{r}}(n_{\mathrm{r}}))$. It leads to the fixed point optimal solution for any initial power allocation. The convergence of sequentially repeating pilot/data power control phases is thus guaranteed and occurs when both $\mathbf{p}_{\mathrm{p}}$ and $\mathbf{p}_{\mathrm{r}}$ cannot be updated within a main loop.

## V. Numerical Results

In the simulation, we consider a $L = 3$ hexagonal cell system with a radius of 1000 meters. Each cell contains $K = 5$ users, which are uniformly placed but at least 100 meters away from the serving BS. For both own and cross channels, i.e. $\mathbf{D}_{li}, \forall(l,i)$, the path-loss exponent is set to $4$ and the large-scale fading is drawn from a log-normal distribution with zero mean and 8 dB standard deviation. We assume that the inter-cell interference of each cell comes exclusively from the other two neighboring cells. The channel coherence interval spans $T = 200$ symbols and the corresponding power budget is equal for all users, i.e. $\forall(l,k), P_{lk} = P$. As we pursue a balanced load among users, the same target SINR $\gamma_{lk} = \gamma, \forall(l,k)$ is applied. In the uplink power control, the initial pilot and data powers are determined as $\forall(l,k), K p_{\mathrm{p},lk} = \frac{1}{6}P$ and $(T-K)p_{\mathrm{r},lk} = \frac{1}{2}P$ [7].

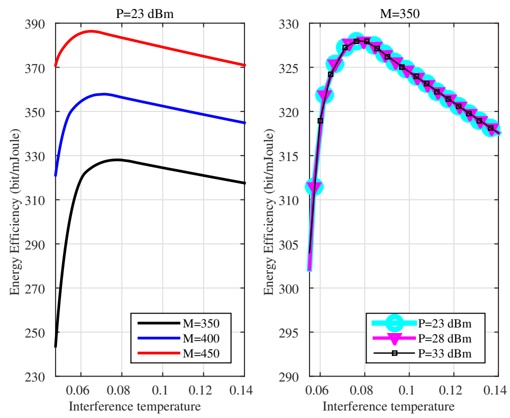


Fig. 1. EE with $\gamma = -5$ dB vs. interference temperature.

First of all, the impact of interference temperature (cf. (10)) on the uplink system EE maximization is investigated. As shown in Fig. 1, the obtained EE is concave w.r.t. the interference temperature for different numbers of BS antennas $M$ and power budgets $P$, thus the optimal interference temperature $I^{\mathtt{opt}}$ can be always acquired via efficient off-line search [5]. In addition, the value of $I^{\mathtt{opt}}$ is insensitive to the total power limit (as extra power is clipped to maximize EE), we assume hence for the rest of simulations that $P = 23$ dBm.

As shown in Fig. 2, the proposed power allocation is by design superior than the power control in regard to the

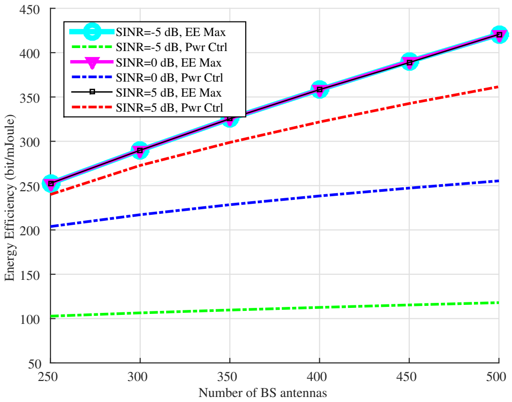


Fig. 2. EE with different $\gamma$ vs. number of BS antennas: "EE Max" denotes the result of proposed power allocation based on $I^{\mathtt{opt}}$ in game $\mathcal{G}_I^{\mathtt{MRC}}$.

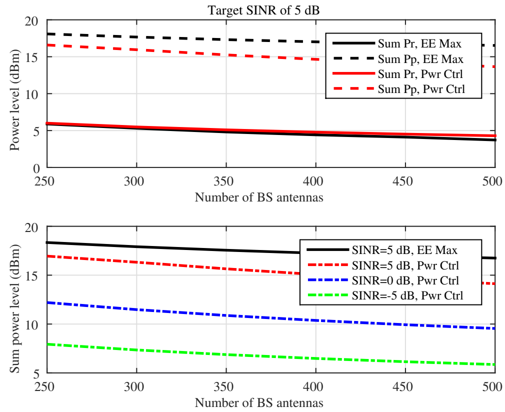


Fig. 3. Power consumption with different $\gamma$ vs. number of BS antennas: pilot power, data power and sum power are shown separately.

TABLE I. ACHIEVED INDIVIDUAL SINR (IN dB) FOR $M = 300$.

| Algo. | Target SINR | Max SINR | Min SINR | Average SINR |
|---|---|---|---|---|
| EE Max | $-5$<br>$0$<br>$5$ | $5.64$ | $5.49$ | $5.57$ |
| Pwr Ctrl | $-5$<br>$0$<br>$5$ | $-5$<br>$0$<br>$5$ | $-5$<br>$0$<br>$5$ | $-5$<br>$0$<br>$5$ |

EE maximization, and the achieved optimal EE values are insensitive to the target SINR, since the optimal solution of (14) lies in the same interior point in spite of different feasible sets confined by the given target SINR constraints[3,4]. Due to the same reason, the obtained SINRs (e.g. as listed in Tab. I for $M = 300$) are also identical for different target SINR settings. On the other hand, the power control attains better EE performance when the target SINR is increased, as it minimizes the sum power by forcing all users to attain exactly the given SINR target (see Tab. I), the EE is then increased for the increment of sum consumed power (see the lower part of Fig. 3) being less than the growth in throughput.

Moreover, as depicted in the upper part of Fig. 3, for both algorithms the optimal assigned power for pilot transmission is much larger than the data power, since only the data transmission benefits from massive BS antennas according to the power-scaling law, whereas the uplink training has to be carried out on a per-antenna basis [2]. Nevertheless, as verified in both figures, deploying more antennas at the BS leads to less sum power consumption and better EE results, which suggests that massive MIMO combined with the proposed power allocation and/or power control can be considered as a key technology component for energy-efficient communications.

## VI. Conclusion

In this paper we have proposed a joint uplink pilot and data power allocation algorithm with MRC receiver in multi-cell MU-Massive-MIMO systems. The algorithm is based on non-cooperative game theory, in which the uplink system EE (with modifications) is maximized via efficient fractional programming. The proposed algorithm has been evaluated in simulations, where the resulting EE distinctly outperforms the uplink power control scheme. Moreover, the EE improvement of joint power allocation becomes more prominent with increased number of BS antennas, thus indicates the advantage of deploying massive MIMO in energy-efficient communications.

[3]Only 1 global optimal solution is reached regardless of the initial powers, which indicates the uniqueness of NE in game $\mathcal{G}_I^{\mathtt{MRC}}$ in the settings considered.

[4]The comparison to the social optimal of game $\mathcal{G}^{\mathtt{MRC}}$ is out of scope, as we focus on the EE maximization algorithm, which is applicable in massive MIMO in practice, and its advantage over the existing power control scheme.

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