Performance of Asymmetric QPSK Modulation for Multi-Level ACK/NACK in LTE Uplink

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***Abstract*—Asymmetric modulation schemes are candidates for conveying quality feedback in the form of multi-level ACK/NACK in a system employing reliability-based HARQ. Asymmetric QPSK (A-QPSK) modulation, for example, enables to convey one ACK level and three NACK levels with unequal error protection of the ACK and NACK levels, each NACK level representing a certain quality level of a received packet. In this work, the performance of A-QPSK modulation for conveying multi-level ACK/NACK is assessed by deriving approximations of the error rates for AWGN channel and by means of link level simulations for LTE uplink. It was found that use of A-QPSK modulation for conveying multi-level ACK/NACK provides valuable transmit power savings over the conventional symmetric QPSK modulation, with gains of about 0.6 – 0.8dB for users at cell edge and about 1.6 – 1.9dB for users with moderate channel conditions. This translates into extra power requirements over BPSK-modulated binary ACK/NACK of about 2.3 – 2.5dB and 1.6 – 1.8dB, respectively.**

I. Introduction

Reliability-based Hybrid ARQ (HARQ) uses quality information computed by the receiver to adapt the format of a retransmission in response to an incorrectly received packet. It has potential to enhance spectral and energy efficiencies of packet transmissions, in particular with imperfect link adaptation caused by inaccurate or outdated channel state information at the transmitter. In the HARQ scheme proposed by Shea [1], a set of least reliably received bits of a packet is retransmitted, which requires to report the respective bit positions to the transmitter. To reduce signaling overhead, the quality information can be used to adapt the *size of a retransmission*, for example, in terms of the number of parity bits or allocated physical resources. With the widely used Incremental Redundancy HARQ [2] scheme, the size of a retransmission is typically adapted by means of puncturing. A HARQ scheme employing multi-level ACK/NACK signaling for conveying control information in addition to a binary ACK/NACK was presented in [3]. When such multi-level ACK/NACK signaling is applied for the quality reporting with reliability-based HARQ, the possible size of a retransmission will be constrained to a small set of values. The actual quality information can be derived by the receiver from the signal-to-noise ratio of the channel [4], or from the soft bits after decoding [5], [6].

Reliability-based HARQ with multi-level ACK/NACK is illustrated in Fig. 1. QPSK modulation with one ACK level and three NACK levels (NACK$_i$, $i \in \{1, 2, 3\}$) provides three possible sizes for a retransmission, e.g. $1.0\times$, $0.5\times$ and $0.25\times$ the size of the initially transmitted packet. The performance

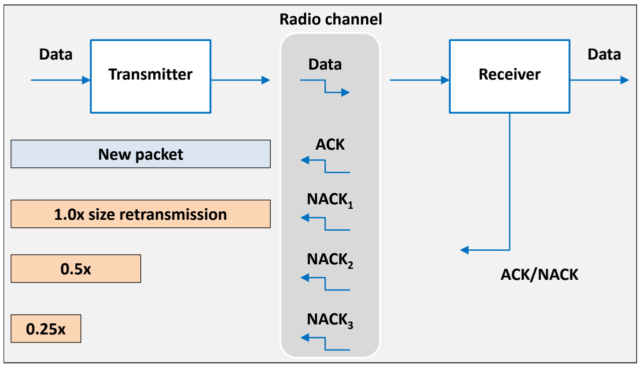

Fig. 1. Reliability-based HARQ transmission scheme employing quality reporting by means of multi-level ACK/NACK.

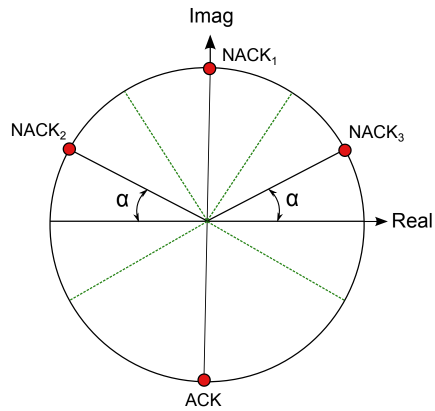

Fig. 2. A-QPSK symbol constellation and decision thresholds (dashed).

of reliability-based HARQ when applied in the 3GPP Long Term Evolution (LTE) system was assessed in [6] by means of link level simulations. It was shown in [6] that a) reliability-based HARQ has potential to improve the data throughput performance over the conventional HARQ schemes used in LTE, b) multi-level ACK/NACK limiting the retransmission size to three values leads to marginal degradation of data throughput performance, and c) NACK $\rightarrow$ ACK transmission errors degrade data throughput more severely than NACK$_i$ $\rightarrow$ NACK$_{j\neq i}$ errors. The latter motivates the application of asymmetric modulation schemes for conveying the multi-level ACK/NACK feedback with unequal error protection for the ACK and NACK levels, respectively.

Fig. 2 exemplifies an asymmetric QPSK (A-QPSK) modulation scheme characterized by the parameter α. A symmetric QPSK modulation is obtained by $\alpha = 0$, and increasing α reduces the NACK $\rightarrow$ ACK transmission error rate while increasing the $\text{NACK}_i \rightarrow \text{NACK}_{j\neq i}$ error rate. For the HARQ protocol to work flawlessly, typically a NACK $\rightarrow$ ACK transmission error rate below $10^{-4} \ldots 10^{-2}$ is required [7], in particular to avoid retransmissions on the higher protocol layers. Using an asymmetric modulation scheme for transmission of multi-level ACK/NACK allows to reduce the ACK/NACK transmit power as compared to using a symmetric modulation scheme, while achieving equally low NACK $\rightarrow$ ACK transmission error rate. This power saving is achieved at the expense of increased $\text{NACK}_i \rightarrow \text{NACK}_{j\neq i}$ error rate, but, as shown in [6], $\text{NACK}_i \rightarrow \text{NACK}_{j\neq i}$ error rates below $3\% \ldots 5\%$ can be tolerated with reliability-based HARQ without causing notable degradation in data throughput or packet delay.

In this paper we will assess the error rate performance of such A-QPSK modulation schemes by means of numerical analysis and link level simulations. A major objective is to quantify the potential for transmit power savings versus the conventional symmetric QPSK modulation scheme for transmission of multi-level ACK/NACK under the following error rate requirements:

$$\Pr\{\text{NACK} \rightarrow \text{ACK}\} \leq \Pi_{\text{NA}}, \quad (1)$$

and

$$\Pr\{\text{NACK}_i \rightarrow \text{NACK}_{j\neq i}\} \leq \Pi_{\text{NN}}, \quad (2)$$

where $\Pr\{.\}$ denotes the probability of occurrence of the error event, and Π_{NA} and Π_{NN} denote some thresholds. Let $\Gamma_{\text{NA}}(\alpha)$ and $\Gamma_{\text{NN}}(\alpha)$ denote the signal-to-noise ratios (SNR) required to satisfy the error rate requirements (1) and (2), respectively. Then the SNR needed to jointly satisfy both these requirements is given by

$$\Gamma(\alpha) = \max\left\{\Gamma_{\text{NA}}(\alpha), \Gamma_{\text{NN}}(\alpha)\right\}, \quad (3)$$

and the power saving of A-QPSK over QPSK is given by

$$\Gamma(\alpha = 0) - \Gamma(\alpha). \quad (4)$$

The remainder of this paper is organized as follows. In Section II, A-QPSK performance over AWGN channel is assessed based on approximations of A-QPSK error rates derived in the Annex. Performance analysis by means of link level simulations for LTE uplink is presented in Section III, and conclusions are drawn in Section IV.

II. AWGN Performance

In this section we assess the performance of A-QPSK over AWGN channel. As derived in the Annex, the NACK $\rightarrow$ ACK error rate can be approximated as

$$\Pr\{\text{NACK} \rightarrow \text{ACK}\} \simeq$$

$$\simeq \frac{P_2 + P_3}{2} \text{erfc}\left(\sqrt{\frac{E}{N_0}} \sin\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right), \quad (5)$$

and the $\text{NACK}_i \rightarrow \text{NACK}_{j\neq i}$ error rate can be approximated as

$$\Pr\{\text{NACK}_i \rightarrow \text{NACK}_{j\neq i}\} \simeq$$

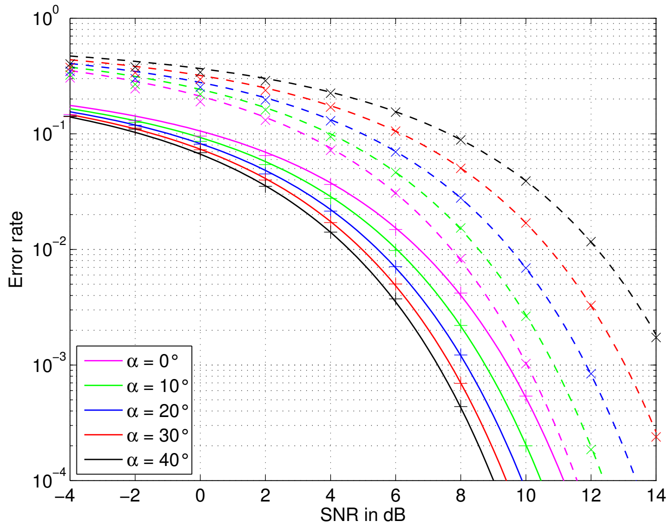

Fig. 3. NACK $\rightarrow$ ACK (solid, '+') and $\text{NACK}_i \rightarrow \text{NACK}_{j\neq i}$ (dashed, 'x') error rates with A-QPSK over AWGN channel, approximated (lines) and simulated ('+', 'x').

$$\simeq \left(P_1 + \frac{P_2 + P_3}{2}\right) \text{erfc}\left(\sqrt{\frac{E}{N_0}} \sin\left(\frac{\pi}{4} - \frac{\alpha}{2}\right)\right), \quad (6)$$

where P_i denotes the *a priori* probability of NACK_i, $\text{erfc}(x)$ denotes the complementary error function, and E/N_0 denotes the SNR.

As the A-QPSK error rates (5) and (6) depend on the *a priori* probabilities P_i, we define the following cases:

- Case 1: $P_1 = P_2 = P_3$ (uniform),
- Case 2: $P_1 = 2P_2 = 2P_3 = 1/2$ (NACK_1 dominant),
- Case 3: $2P_1 = P_2 = P_3 = 2/5$ ($\text{NACK}_{2,3}$ dominant).

For Case 1, the error rates (5) and (6) are illustrated in Fig. 3 by solid and dashed lines, respectively, and respective error rates obtained by means of computer simulations for AWGN channel are indicated by '+' and 'x'. We observe that the $\text{NACK}_i \rightarrow \text{NACK}_{j\neq i}$ error rate is more sensitive to increases of α than the NACK $\rightarrow$ ACK error rate. Within the relevant value range we find approximations (5) and (6) tightly aligned with the simulated error rates for the considered values of α. This motivates to numerically compute the power saving of A-QPSK over QPSK from approximations (5) and (6).

For Case 1, the power saving of A-QPSK over QPSK while satisfying the error rate requirements (1) and (2) is depicted in Fig. 4 as a function of the angle α with values of Π_{NA} between $0.01\% \ldots 1\%$, and with $\Pi_{\text{NN}} = 5\%$ (top) and $\Pi_{\text{NN}} = 1\%$ (bottom). The respective maximum power savings and optimum values of α are summarized in Table I. It can be seen that tightening the Π_{NA} threshold increases the maximum achievable transmit power saving over QPSK by enabling larger values of α, and similar trends are found for relaxing the Π_{NN} threshold. With $\Pi_{\text{NA}} = 0.1\%$ and $\Pi_{\text{NN}} = 5\%$, for example, a transmit power saving over QPSK of up to about 1.67dB is obtained with A-QPSK modulation using $\alpha = 28.1°$.

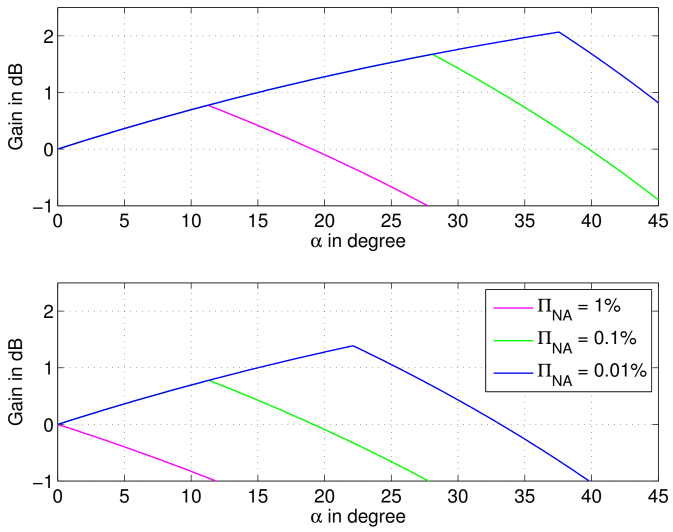

Fig. 4. Power saving of A-QPSK over QPSK versus α for Case 1 with thresholds $\Pi_{\mathrm{NA}} = 0.01\%$, 0.1% and 1%, and thresholds $\Pi_{\mathrm{NN}} = 5\%$ (top) and $\Pi_{\mathrm{NN}} = 1\%$ (bottom).

TABLE I. MAXIMUM TRANSMIT POWER SAVINGS OVER QPSK AND OPTIMUM VALUES OF α VERSUS THRESHOLDS Π_{NA} AND Π_{NN} (CASE 1).

	Π_{NN}	$\Pi_{\mathrm{NA}} = 0.01\%$	$\Pi_{\mathrm{NA}} = 0.1\%$	$\Pi_{\mathrm{NA}} = 1\%$
max. gain	5%	2.07dB	1.67dB	0.77dB
optimum α		37.6°	28.1°	11.3°
max. gain	3%	1.85dB	1.38dB	0.33dB
optimum α		32.0°	21.9°	4.5°
max. gain	1%	1.39dB	0.78dB	0dB
optimum α		22.1°	11.3°	0°

The power saving of A-QPSK over QPSK while satisfying the error rate requirements (1) and (2) is depicted in Fig. 5 for Cases $1-3$ as a function of the angle α, assuming $\Pi_{\mathrm{NN}} = 5\%$ and $\Pi_{\mathrm{NA}} = 0.01\%$, 0.1% and 1%. It can be seen that the maximum power saving is highest with Case 3 and lowest with Case 2. When tightening Π_{NA} to values $< 1\%$ the impact of the *a priori* probabilities P_i on the power saving becomes less pronounced. We further find that power savings are obtained in all Cases $1-3$ if α remains below a certain threshold, for example, if $\alpha < 11.7^\circ$ with $\Pi_{\mathrm{NA}} = 1\%$.

The results of Figs. 4 and 5 were numerically derived from the error rate approximations (5) and (6) by computing $\Gamma_{\mathrm{NA}}(\alpha)$ and $\Gamma_{\mathrm{NN}}(\alpha)$ with 0.01° resolution of α and 0.002 dB resolution of the SNR (without applying interpolation).

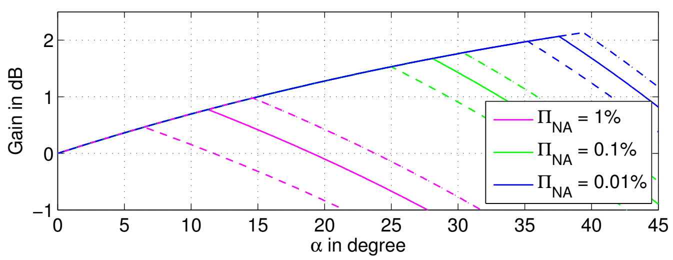

Fig. 5. Power saving of A-QPSK over QPSK versus α for Case 1 (solid), Case 2 (dashed) and Case 3 (dashdot), assuming $\Pi_{\mathrm{NN}} = 5\%$ and $\Pi_{\mathrm{NA}} = 0.01\%$, 0.1% and 1%.

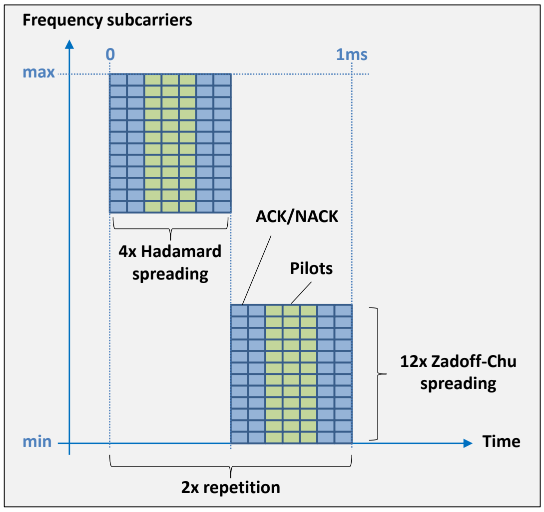

Fig. 6. ACK/NACK transmission on LTE PUCCH.

III. LTE UPLINK PERFORMANCE

In this section we present link level simulation results to assess the performance of A-QPSK modulation when applied to LTE Physical Uplink Control Channel (PUCCH) [8].

A. Simulation Setup

A single-cell link level simulator for LTE uplink is used, which includes a realistic modeling of fading channels and signal processing algorithms such as modulation, coding, spreading, physical resource mapping and channel estimation. We assume single antenna transmission with OFDM modulation, 10MHz system bandwidth at 2.3GHz carrier frequency and two receive antennas with uncorrelated fading channels.

We assume that a NACK_i level is transmitted in each 1ms subframe, the NACK_i being uniformly drawn from the set NACK_i, $i \in \{1, 2, 3\}$ (as in Case 1 above), and being mapped to an A-QPSK modulation symbol as illustrated in Fig. 2. The modulation symbols are spread in the frequency domain by Zadoff-Chu sequences of length 12, spread in the time domain by Hadamard sequences of length 4, repeated in time by $2\times$, and mapped to the OFDM resource elements of the PUCCH together with demodulation pilot symbols, as illustrated in Fig. 6, with frequency hopping at the 0.5ms slot boundary [8]. The receiver performs channel estimation using the demodulation pilot symbols and performs maximum ratio combining of the two received signals. The receiver then applies decision thresholds as indicated in Fig. 2 and decides for the modulation symbol being closest in angle to the received symbol. NACK $\rightarrow$ ACK and $\mathrm{NACK}_i \rightarrow \mathrm{NACK}_{j \neq i}$ error rates are computed.

B. Simulation Results

Link level simulations with A-QPSK modulation were performed with realistic channel estimation, for AWGN channel and for a variety of fading channels. The error rate requirement assumed for quantifying the power saving of A-QPSK over QPSK is defined by (1) and (2) while setting the thresholds according to

$$\Pi_{\mathrm{NA}} = 0.1\% \quad \& \quad \Pi_{\mathrm{NN}} = 5\%. \tag{7}$$

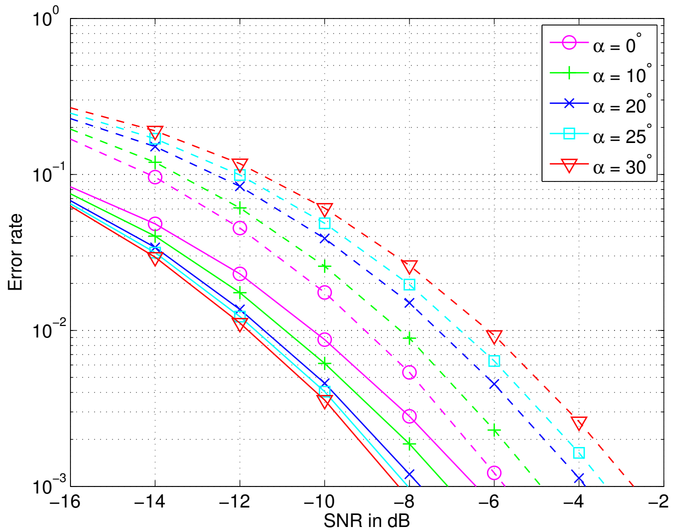

Fig. 7. Simulated NACK $\rightarrow$ ACK (solid) and $\text{NACK}_i \rightarrow \text{NACK}_{j\neq i}$ (dashed) error rates with A-QPSK over ePedA 3km/h channel.

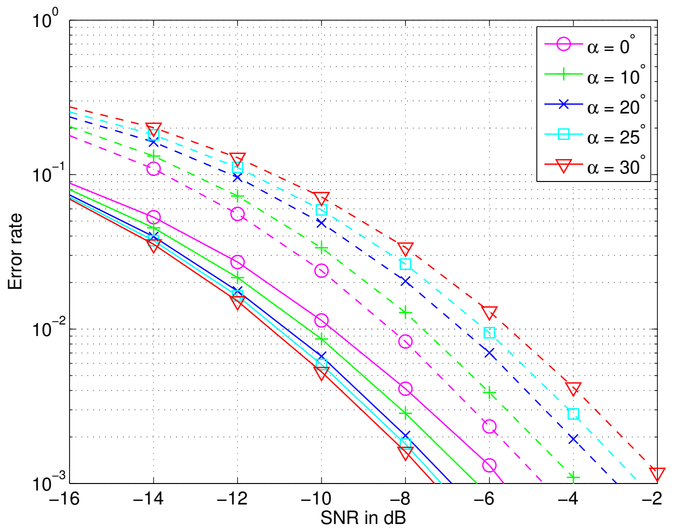

Fig. 8. Simulated NACK $\rightarrow$ ACK (solid) and $\text{NACK}_i \rightarrow \text{NACK}_{j\neq i}$ (dashed) error rates with A-QPSK over VehA 30km/h channel.

Motivated by Fig. 5, simulations were performed with $\alpha \leq 30°$.

The simulated error rates versus SNR are depicted in Figs. 7 and 8 for ePedA 3km/h and VehA 30km/h channels, respectively. Due to the coding/spreading gain of LTE PUCCH of about 19.8dB, ACK/NACK can reliably be transmitted at negative SNRs $\ll$ 0dB. We observe that the $\text{NACK}_i \rightarrow \text{NACK}_{j\neq i}$ error rate is more sensitive to increases of α than the NACK $\rightarrow$ ACK error rate, similar as with AWGN in Fig. 3. We further observed that realistic channel estimation degrades error rate performance by about 4dB at such negative SNRs as compared to ideal channel estimation.

The simulated SNR values $\Gamma(\alpha)$ needed to satisfy the error rate requirement (7) when conveying multi-level ACK/NACK with A-QPSK modulation are listed in Table II in dB units for different channel profiles and values of α. For AWGN channel profile, these SNR values are determined by the $\text{NACK}_i \rightarrow \text{NACK}_{j\neq i}$ error rate requirement (2) when α exceeds about 25°, while for the simulated multi-path fading channels these SNR values are determined alone by the NACK $\rightarrow$ ACK error rate requirement (1) over the simulated range of α. Table II further contains the simulated SNR values needed to satisfy the NACK $\rightarrow$ ACK error rate requirement (1), when conveying binary ACK/NACK with BPSK modulation.

TABLE II. Required SNR for BPSK and A-QPSK modulation with different channel profiles, in dB units.

	BPSK	$\alpha = 0°$	$\alpha = 10°$	$\alpha = 20°$	$\alpha = 30°$
AWGN	-13.25	-9.80	-10.55	-11.15	-11.70
ePedA 3km/h	-9.90	-6.40	-7.05	-7.70	-8.25
VehA 30km/h	-9.10	-5.65	-6.25	-6.85	-7.30

TABLE III. Power saving of A-QPSK relative to QPSK ($\alpha = 0°$) modulation, in dB units.

	$\alpha = 10°$	$\alpha = 20°$	$\alpha = 25°$	$\alpha = 30°$
AWGN	0.75	1.35	1.60	1.90
ePedA 3km/h	0.65	1.30	1.60	1.85
VehA 30km/h	0.60	1.20	1.45	1.65

The power savings of A-QPSK relative to QPSK ($\alpha = 0°$) in order to satisfy the error rate requirement (7) are listed in Table III in dB units. For AWGN channel profile, it can be seen that the power saving of A-QPSK over QPSK ($\alpha = 0°$) modulation is similar to the results of Case 1 in Fig. 5, although the gains and the optimum value of α are a bit higher. In the presence of multi-path fading, the simulated transmit power savings over QPSK appear similar as in the presence of AWGN. It can be seen that A-QPSK provides transmit power savings versus QPSK of about $1.6 - 1.9$dB for $\alpha = 30°$ with the above channel profiles. Fine-tuning of α beyond $\alpha = 30°$ may increase the power saving somewhat in the presence of fading, whereas no further gains are expected with AWGN. In the presence of fading, the power savings of Table III are also achievable when tightening the $\text{NACK}_i \rightarrow \text{NACK}_{j\neq i}$ error rate requirement (2) to $\Pi_{\text{NN}} \leq 3\%$.

The extra power needed to convey A-QPSK-modulated multi-level ACK/NACK relative to BPSK-modulated binary ACK/NACK while satisfying the respective error rate requirements are listed in Table IV in dB units. It can be seen that with $\alpha = 30°$ the extra power required for multi-level ACK/NACK is about $1.6 - 1.8$dB as compared to BPSK-modulated binary ACK/NACK with the simulated channels.

We similarly quantified the power saving of A-QPSK over QPSK when relaxing the NACK $\rightarrow$ ACK error rate requirement (1) to $\Pi_{\text{NA}} \leq 1\%$, while keeping the $\text{NACK}_i \rightarrow \text{NACK}_{j\neq i}$ error rate requirement (2) at $\Pi_{\text{NN}} \leq 5\%$. In this case, a power saving over QPSK of about $0.6 - 0.8$dB can be realized by using $\alpha = 10° \ldots 15°$, which requires an extra power over BPSK of about $2.3 - 2.5$dB.

The above simulation results for LTE PUCCH assume single user transmission and perfect time and frequency syn-

TABLE IV. Extra power of A-QPSK relative to BPSK modulation, in dB units.

	$\alpha = 0°$	$\alpha = 10°$	$\alpha = 20°$	$\alpha = 25°$	$\alpha = 30°$
AWGN	3.45	2.70	2.10	1.85	1.55
ePedA3	3.50	2.85	2.20	1.90	1.65
VehA30	3.45	2.85	2.25	2.00	1.80

chronization. We also carried out respective simulations for a single cell with multiple users transmitting ACK/NACK in code multiplex, and we found that the presence of multi-user interference can be neglected in the results. This is due to the close-to-perfect orthogonality of the spreading sequences used on PUCCH. Further, we found in simulations that frequency offsets due to oscillator mismatch can be neglected, as such offsets are small compared to the 15kHz subcarrier spacing of LTE.

IV. Conclusion

The performance of asymmetric QPSK (A-QPSK) modulation, characterized by the constellation angle α as shown in Fig. 2, for conveying multi-level ACK/NACK with three NACK levels NACK_i was assessed. Multi-level ACK/NACK can be applied to adapt the size of a retransmission with reliability-based HARQ. We assumed a NACK $\rightarrow$ ACK transmission error rate requirement below a first threshold $\Pi_{\text{NA}} = 0.01\% \ldots 1\%$ [7] and a $\text{NACK}_i \rightarrow \text{NACK}_{j \neq i}$ error rate requirement below a second threshold $\Pi_{\text{NN}} = 3\% \ldots 5\%$ [6]. A major objective was to quantify the potential for transmit power savings over the conventional symmetric QPSK modulation scheme, while satisfying the above error rate requirements.

For AWGN channel, approximations of the error rate performance of A-QPSK for conveying multi-level ACK/NACK were derived, by means of which the power saving of A-QPSK over QPSK was computed numerically. It was found that the maximum achievable power saving and the respective angle α increase with tightening the Π_{NA} threshold or with relaxing the Π_{NN} threshold, for example, a gain of up to 1.67dB with $\alpha = 28.1°$ was found in the presence of AWGN with $\Pi_{\text{NA}} = 0.1\%$ and $\Pi_{\text{NN}} = 5\%$. The dependency of the power saving on the achievable error rates motivates to adapt the A-QPSK constellation angle α to the SNR, where α shall be increased with increasing SNR. Possibly, A-QPSK-modulated multi-level ACK/NACK may be used at moderate-to-high SNR, while at low SNR BPSK-modulated binary ACK/NACK may be used to extend coverage. As the power saving of A-QPSK over QPSK is dependent also on the *a priori* probabilities of the NACK_i, this further motivates to control the frequency of occurrence of the NACK_i by adequate formatting of the data packets transmitted using the HARQ protocol.

For the LTE system, link level simulations in the presence of multi-path channel fading were performed to assess performance of A-QPSK modulation for conveying multi-level ACK/NACK on the Physical Uplink Control Channel (PUCCH), while assuming equal *a priori* probabilities of the NACK_i. With $\Pi_{\text{NA}} = 0.1\%$, which appears characteristic for users with moderate channel conditions [7], A-QPSK with $\alpha \simeq 30°$ provides transmit power savings over QPSK of about $1.6 - 1.9$dB, while about $1.6 - 1.8$dB extra transmit power relative to BPSK modulated binary ACK/NACK are required. With $\Pi_{\text{NA}} = 1\%$, which appears characteristic for users at cell edge or users moving with high velocity [7], A-QPSK with $\alpha \simeq 10° - 15°$ provides transmit power savings over QPSK of about $0.6-0.8$dB, while about $2.3-2.5$dB extra transmit power relative to BPSK modulated binary ACK/NACK are required.

The above transmit power savings of A-QPSK over QPSK modulation are obtained at the expense of somewhat increased computational complexity for the demodulation, since A-QPSK demodulation involves the application of decision thresholds in polar coordinates as indicated in Fig. 2.

A further aspect to be considered with ACK/NACK detection is the detection of discontinued transmission (DTX), i.e. the ACK/NACK detector should be able to detect if neither ACK nor NACK was transmitted, e.g. if the received signal power is below a DTX detection threshold. DTX may occur if the respective scheduling grant for the data packet could not be correctly decoded. DTX should be decided as a NACK (or NACK_1), and the error probability $\Pr\{\text{DTX} \rightarrow \text{ACK}\}$ of DTX transmitted and ACK received should be small. With A-QPSK-modulated multi-level ACK/NACK, the DTX $\rightarrow$ ACK error probability increases with increasing α by the factor $(1 + 2\alpha/\pi)$, i.e. by a factor of up to about 1.33 for $\alpha \leq 30°$. When comparing A-QPSK-modulated multi-level ACK/NACK versus BPSK-modulated binary ACK/NACK, the ratio of the DTX $\rightarrow$ ACK error probabilities is given by $(0.5 + \alpha/\pi)$, i.e. 0.5 for QPSK ($\alpha = 0$) and 0.67 for $\alpha = 30°$. It can be seen that the order of magnitude of the DTX $\rightarrow$ ACK error probabilities is similar for A-QPSK and QPSK, while for BPSK it is 2x larger than for QPSK. Possible techniques to reduce the DTX $\rightarrow$ ACK error probabilities of A-QPSK are to contain the value of α, or to increase the DTX detection threshold, possibly together with boosting the transmit power of the ACK symbol in Fig. 2.

Possible aspects for further research include the development and performance assessment of algorithms for DTX detection, the assessment of higher order modulation schemes supporting more than three NACK levels, or the assessment of non-unitary modulation schemes, for example, the transmit power of the NACK_1 symbol in Fig. 2 may be boosted in order to reduce the $\text{NACK}_i \rightarrow \text{NACK}_{j \neq i}$ error rate.

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Annex: Derivation of AWGN Performance

Approximations of the A-QPSK error rates can be derived from the symbol error rate of MPSK modulation, which in the presence of AWGN is approximately given by

$$\text{erfc}\left(\sqrt{\frac{E}{N_0}} sin\left(\frac{\pi}{M}\right)\right), \tag{8}$$

where E/N_0 denotes the SNR, M denotes the number of symbols of the symmetric MPSK constellation, and

$$\text{erfc}\,(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt \tag{9}$$

denotes the complementary error function [9]. Approximation (8) holds for moderate-to-high SNRs, and it is determined by the minimum distance between two MPSK modulation symbols given by $2\sqrt{E}\sin(\pi/M)$, where $\sqrt{E}$ denotes the amplitude of the signal constellation points.

In analogy to the MPSK symbol error rate, the $\mathrm{NACK}_1 \rightarrow \mathrm{NACK}_{2,3}$ conditional error rate for A-QPSK can be approximated as

$$\Pr\{\mathrm{NACK}_{2,3} \mid \mathrm{NACK}_1\} \simeq$$

$$\simeq \mathrm{erfc}\left(\sqrt{\frac{E}{N_0}}\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\right), \qquad (10)$$

where

$$2\sqrt{E}\sin\left(\frac{\pi}{4}-\frac{\alpha}{2}\right) \qquad (11)$$

denotes the distance between the NACK_1 and NACK_2 or NACK_3 constellation points. For $\alpha \ll \pi/2$ the distance between the NACK_2 and NACK_3 symbols exceeds the minimum distance (11) and we can approximate

$$\Pr\{\mathrm{NACK}_{1,3} \mid \mathrm{NACK}_2\} \simeq$$

$$\simeq \Pr\{\mathrm{NACK}_{1,2} \mid \mathrm{NACK}_3\} \simeq$$

$$\simeq \frac{1}{2}\Pr\{\mathrm{NACK}_{2,3} \mid \mathrm{NACK}_1\}. \qquad (12)$$

By taking into account the *a priori* probabilities P_i of the NACK_i symbols, we obtain an approximation of the $\mathrm{NACK}_i \rightarrow \mathrm{NACK}_{j\neq i}$ error rate according to (6).

The minimum distance between ACK and NACK symbols of the A-QPSK constellation is given by

$$2\sqrt{E}\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right), \qquad (13)$$

determined by the NACK_2 and NACK_3 symbols, and so we obtain an approximation of the NACK $\rightarrow$ ACK error rate according to (5).

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