Distributed Precoding in Multicell Multiantenna Systems With Data Sharing

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Abstract— In this paper, a precoding scheme for interference mitigation based only on local channel state information is proposed for downlink multicell multiantenna systems with joint transmission. The investigated scheme places interference components at the receivers coming from the different transmitters in predefined subspaces, so that they are completely or approximately cancelled. It is shown how in certain scenarios of practical interest this strategy exploits better the available degrees-of-freedom in the system, with very little or practically no additional coordination effort compared with the related results from the literature.

Keywords— Distributed precoding, joint processing, interference mitigation, imperfect backhaul, multiantenna systems, underdetermined systems, caching.

I. INTRODUCTION

In the domain of multicell MIMO systems, it is known that inter-cell cooperation under idealistic assumptions can significantly improve the system performance [1]. However, in such schemes, the requirements for user data and channel state information (CSI) sharing over a backhaul network often present a limiting factor in practical scenarios [2].

For this reason, in recent years, significant efforts have been invested in developing methods which require less information exchange over the backhaul links. As limited capacity of the backhaul was one of the main problems in the past, one approach of interest was to develop schemes which do not assume user data sharing, but can cooperate using CSI exchange among the base stations. A typical example in this direction is coordinated beamforming (CB) [3][4]. However, CB and similar systems still demand often information exchange among the base stations (BSs) in a very fast manner. This makes in practice the deployment of such systems almost as challenging as the deployment of full joint processing (JP) MIMO systems which utilize both user data and CSI exchange, since low latency backhaul must be ensured.

On the other hand, it is expected that ultra-dense, heterogeneous deployments will play a prominent role in enabling significantly higher data rates in future cellular systems. In other words, we are likely to face versatile network scenarios both in terms of cell size and the backhaul



Fig. 1. Cooperation in a heterogeneous wireless network.

support.

For backhaul solutions, in recent years there have been significant advances in the work on 100 Gb/s optical Ethernet [5], passive optical networks (PON) [6], and microwave communication [2]. However, the latency introduced by the backhaul is still quite high in practice if the cooperating BSs must communicate through more aggregation layers in the backhaul or even through core network, especially if the traffic is not prioritized. Further, low capacity and high latency links, such as digital subscriber lines (DSL), are still utilized and might remain a preferred choice in some cases for low-cost small cells in the future. These situations are illustrated in Fig. 1, with dashed lines depicting the links for information exchange for inter-node cooperation in two exemplary cases.

Regarding latency, one should not forget that in addition to the imperfect backhaul, a non-negligible delay is introduced at the BS site itself due to user feedback, channel estimation, signal processing, etc. Therefore, the often assumed fast exchange of

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CSI and alike information might remain a bottleneck factor in future cooperative multicell multiantenna systems.

While the dynamics of wireless channels are associated with physical phenomena one cannot change easily, the user data in many applications have predictable behavior that allows their sharing and cashing/buffering (illustrated with "files" next to the base stations in Fig. 1) at convenient locations in future networks, having in mind that the costs for memory will likely remain well below the costs for building a low latency backhaul network. In other words, although the amount of data to be shared is higher for the actual user data compared with the overhead CSI, in certain scenarios of significant practical interest it might indeed be easier to have the user data available in the cooperating transmitters. With this motivation in mind, we are interested in an extended set of cooperation possibilities. The objective of this research is namely to examine the so far less-understood multicell cooperative techniques that do not exploit CSI exchange (constrained by a short time delay requirement for this exchange), but might still benefit from data sharing or caching. We remark that caching of data in radio access networks (e.g., for media files downloading) has recently attracted a lot of attention in the community [7] [8].

This works differs from most of the contributions in the area of cooperative wireless systems primarily in the assumption on the information exchange model for cooperation. While the classical JP and CB scenarios are and certainly will remain of great interest, as explained above, future wireless systems should support a broader set of cooperation possibilities in order to adapt to the new deployments and consider realistic backhaul latency. One line of research regarding the backhaul latency assumes analysis and compensation of the delays introduced in CSI distribution, often utilizing some inherent wireless channel characteristics such as temporal correlation [9][10]. On the other hand, one might try to optimize the multi-antenna precoders and equalizers by exploiting the locally available CSI (in this paper we focus on time-division duplex (TDD) scenarios, and the meaning of local CSI will be explained in detail in Section II). The scheme investigated here falls into this group (unlike the ones from [8]), and it exploits the supposed possibility of data sharing. The relevant works in the literature for the case we study are [11][12] (cf. also [13]).

Our contributions can be briefly summarized as follows:

- We propose a scheme which conveniently places the interference in the subspaces agreed offline by the BSs, and show how in some practical scenarios of interest all interference components can be added in a destructive manner. For this reason, we coin the investigated scheme (distributed) Destructive Interference Addition (DIA).
- The DIA method is shown to admit more users in the system than zero forcing (ZF) approaches (cf., e.g., [14]), while being able to keep the available degrees-of-freedom (DoFs).



Fig. 2. Simplified system model with single-antenna users.

• By preserving the DoFs, the interference-free solution and its variation that we study, outperform significantly the referent works [11][12] in many relevant cases, while little or practically no additional cooperation effort is introduced. Further, unlike in the related works, the method proposed here is applicable for multi-stream transmission to multi-antenna users, as well.

A. Notation

We use small and large bold fonts to denote vectors and matrices, respectively. The dimensions, if not explicitly stated, will be clear from the context. I is the identity matrix, and 0 is the zero-matrix. The Frobenius norm is denoted by $\|(\cdot)\|_F$. The transpose and conjugate (Hermitian) transpose of a matrix are written as $(\cdot)^T$ and $(\cdot)^H$, respectively. A^{\dagger} is the Moore–Penrose pseudoinverse of A [15]. C^N is the *N*-dimensional complex space, and $E(\cdot)$ is the expectation operator.

B. Outline of the Paper

The rest of the paper is organized as follows. In Section II, the basic system model with single-antenna users and the main problem statement are presented. Section III describes the proposed DIA scheme in two variations with respect to the coordination effort and power control, and explains its application also to a more general system model with multi-antenna users. Numerical examples followed by a discussion of results are shown in Section IV, and some concluding remarks are given in Section V.

II. SYSTEM MODEL AND PROBLEM STATEMENT

A simplified system model under study is shown in Fig. 2. We consider downlink transmission in a multicell multiple-input single-output (MISO) system, on one shared resource (flat fading scenario). There are M BSs in the system transmitting simultaneously to K mobile stations (MSs). For convenience, each BS is assumed to be equipped with N antennas, while the mobiles have single antennas. We initially describe our approach using this case as it reveals already many interesting

aspects, and as it enables a direct comparison with the related approaches from the literature, which handle single stream transmission per user. We remark, however, that the principal idea is directly applicable to the general multicell multiuser MIMO setup, where BSs and MSs have arbitrary number of antennas, and where more data streams per MS are transmitted. Some comments for this general case are given in Section III.A. Further, our particular interest is in less-understood, underdetermined, distributed systems where the relation N < K holds, though the scheme we analyze is applicable for $N \ge K$, as well. In other words, we try to maximize the number of active users which can be supported in a (nearly) interference-free manner.

The flat fading channel between the BS_m and the MS_k is denoted by $\mathbf{h}_{mk} \in \mathbb{C}^N$. We suppose that each base station has only local transmit CSI. In other words, the BS_m knows the channels \mathbf{h}_{mk} , k=1,...,K, perfectly, and it has no information about the channels collected at other base stations. Such scenario matches well TDD setups. For notational convenience, all channels at the BS_m are grouped into one matrix $\mathbf{H}_m \triangleq [\mathbf{h}_{m1} \cdots \mathbf{h}_{mK}]$.

In this paper, we analyze the case where all user data is assumed to be shared among the cooperating base stations. At one time instant, the symbol intended for the MS_k is x_{k_i} and the complete data vector $\mathbf{x} = [x_1 \ \cdots \ x_K]^T$, with $E\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}$, is available to all BSs.

Let the linear precoder of the BS_m for the MS_k be denoted by $f_k^{(m)}(H_m) \in C^N$, where the notation with the channel in brackets is used to emphasize the fact that the precoder is a function of the local channel knowledge only, and will be implicitly assumed and occasionally omitted in the sequel. The signal transmitted from the BS_m denoted by s_m , is then

$$s_m = \sum_{k=1}^{K} f_k^{(m)} x_k.$$
 (1)

The signal that the MS_k receives is given as

$$y_k = \sum_{m=1}^{M} \boldsymbol{h}_{mk}^H \, \boldsymbol{s}_m + \, n_k, \quad k = 1, \dots, K,$$
 (2)

where n_k is the complex white additive noise with $E\{|n_k|^2\} = \sigma^2, \forall k$. Clearly, the scheme at hand can be seen as a superposition of *M* multiuser MISO channels [12], which are well-understood separately (cf., e.g., [16]).

Based on (1) and (2), the signal-to-interference-plus-noise ratio (SINR) at the MS_k (assumed to have perfect CSI) is

$$SINR_{k} = \frac{\left|\sum_{m=1}^{M} \boldsymbol{h}_{mk}^{H} \boldsymbol{f}_{k}^{(m)}\right|^{2}}{\sum_{\substack{l=1\\l\neq k}}^{K} \left|\sum_{m=1}^{M} \boldsymbol{h}_{mk}^{H} \boldsymbol{f}_{l}^{(m)}\right|^{2} + \sigma^{2}}.$$
 (3)

The problem of interest will be the sum-rate maximization in the system

$$\max_{\substack{\{f_l^{(m)}(H_m)\}\\m=1,\dots,M\\l=1,\dots,K}} R \triangleq \sum_{k=1}^{K} \log (1 + \text{SINR}_k)$$
(4)
subject to $\|f_1^{(m)} \cdots f_K^{(m)}\|_F^2 \leq P_{tot}, \quad \forall m,$

where P_{tot} is the per-BS power constraint, assumed to be the same for all BSs w.l.o.g.

III. DESTRUCTIVE INTERFERENCE ADDITION

In this contribution, we analyze an interference-free solution at the reception for the system model shown in Section II. From the SINR expression (3), it can be noticed that the problem (4) is very involved, particularly if it is to be solved in a distributed way. Namely, the precoders in the cooperating cluster, dependant on the channels collected at different BSs, are coupled both in the numerator and the denominator of the users' SINR expressions. From (3), it can be seen that for the interference-free reception at MS_k , the following condition is to be satisfied

$$\sum_{m=1}^{M} \boldsymbol{h}_{mk}^{H} \boldsymbol{f}_{l}^{(m)} = \boldsymbol{0}, \quad \forall l \in \{1, \dots, K\} \setminus \{k\}.$$
(5)

We show in the sequel how the precoders of the BSs can be calculated to satisfy the interference-free condition in certain scenarios using the local CSI only.

Let $\boldsymbol{v}_1, ..., \boldsymbol{v}_M$ denote a predefined arbitrary set of fixed C^{K-1} vectors known to all BSs, with

$$\boldsymbol{v}_1 + \dots + \boldsymbol{v}_M = \boldsymbol{0}. \tag{6}$$

We will discuss the possibilities for selecting convenient $v_1, ..., v_M$ in Section IV. It can be easily checked that the interference free condition for all MSs is satisfied if

$$\begin{bmatrix} \boldsymbol{h}_{m1}^{H} \\ \vdots \\ \boldsymbol{h}_{m,k-1}^{H} \\ \boldsymbol{h}_{m,k+1}^{H} \\ \vdots \\ \boldsymbol{h}_{mK}^{H} \end{bmatrix} \boldsymbol{f}_{k}^{(m)} = \boldsymbol{v}_{m}, \ \forall k = 1, \dots, K, \ \forall m = 1, \dots, M.$$
(7)

The above given sufficient condition for calculating the precoder $f_k^{(m)}$ of the BS_m for the MS_k depends only on the locally available channels at the BS_m. As the fixed vector v_m is known to the BS_m in advance, there exists practically no additional effort in information exchange for cooperation. The equation (7) is solvable (for generic channels, i.e., for almost all channel realizations [17]) if

$$N \ge K - 1. \tag{8}$$

From (8), we see a possibility of handling some underdetermined configurations, as well (for an even larger number of users, the scheme should be combined with some scheduling strategies). In other words, an additional user can be supported by applying the proposed approach, compared to the (distributed) zero-forcing (ZF) method. A solution for $f_k^{(m)}$ in this case is

$$\boldsymbol{f}_{k}^{(m)} = \begin{bmatrix} \boldsymbol{h}_{m1}^{H} \\ \vdots \\ \boldsymbol{h}_{mk-1}^{H} \\ \boldsymbol{h}_{mk+1}^{H} \\ \vdots \\ \boldsymbol{h}_{mK}^{H} \end{bmatrix}^{\dagger} \boldsymbol{v}_{m}, \ \forall k = 1, \dots, K, \ \forall m = 1, \dots, M.$$
(9)

As the precoders calculated from (9) can violate the power constraints of (4), we present two approaches which scale the solutions so that these constraints are satisfied:

- *DIA*: In this method, all precoders $f_k^{(m)}$, k = 1, ..., K, m = 1, ..., M, are scaled with the same positive real factor, so that the most critical per-BS power constraint is satisfied. Notice that this requires additional exchange of information of one scalar value among the BSs and the distribution of the common scaling factor, which must be realized in a fast manner.
- *DIA with uncoordinated power control (DIA-UPC)*: In this case, the precoders $f_k^{(m)}$, k = 1, ..., K, at the BS_m are scaled with a positive scalar α_m so that

$$\left\|\alpha_m [\boldsymbol{f}_1^{(m)} \quad \cdots \quad \boldsymbol{f}_K^{(m)}]\right\|_F = \sqrt{P_{tot}}.$$
 (10)

Clearly, this can be done in a completely distributed way.

We note that the application of both schemes is possible with straightforward modifications also in scenarios with arbitrary per-BS or per-antenna power constraints.

An illustration of the DIA and DIA-UPC approaches is given in Fig. 3 for the case of 2 BSs equipped with 2 antennas and 3 single-antenna users.

In this case, for the DIA scheme, the BS₁ is designing its precoder (beamformer) $f_1^{(1)}$ so that the interference factors $[h_{12}^H f_1^{(1)} h_{13}^H f_1^{(1)}]^T$ match the vector $v_1 = v$. Other precoders of the BS₁ are calculated in an analogous way. The BS₂ is performing the same strategy, but using $v_2 = -v$, so that the interference components are completely cancelled (possibly after scaling with the same factor to satisfy the power constraints, as illustrated by the blue and red full vectors in Fig. 3). For the DIA-UPC, the interference factors from the BS₁ and BS₂ have only the same direction as v_1 and v_2 , respectively, while the interference amplitudes can be different after scaling, as illustrated by the dashed vectors. In other words, for the DIA-UPC, the interference is only approximately cancelled.



Fig. 3. Illustration of the DIA method.

We note that, as in the case described by Fig. 3, N = K - 1 holds, no further optimization of the factors multiplying the desired signals is done.

Interestingly, from Fig. 3 it can be seen that both DIA and DIA-UPC perform a kind of interference alignment. However, one should notice that the assumptions on the system model are very different from the ones in [17].

A. Multi-Antenna Users

In order to simplify the exposition, let each MS be equipped with *R* antennas, and receive *D* data streams (layers). The flat fading channel between the BS_m and MS_k is written as $H_{mk} \in C^{R \times N}$. The data for the mobile station (user) MS_k, are grouped into the vector $x_k \in C^D$. The complete data vector xis then the concatenation of all user data vectors. The precoder of the BS_m for the MS_k is now a matrix $F_k^{(m)} \in C^{N \times D}$. We assume that the MS_k is equipped with a linear equalizer $G_k \in C^{D \times R}$. For comparison purpose, the goal in the MIMO case will be analogous to the problem (4), with the objective function defined as the sum rate of the layers of all MSs

$$R = \sum_{k=1}^{K} \sum_{d=1}^{D} \log(1 + \text{SINR}_{k,d}),$$
(11)

where $\text{SINR}_{k,d}$ is the post-processing SINR of the *d* th layer of the MS_k. We remark that higher rates can be achieved with optimized processing of the receive data streams at one user.

The interference-free solution at the reception in this case has to satisfy the following condition: M

$$\boldsymbol{G}_{k} \sum_{m=1}^{m} \boldsymbol{H}_{mk} \boldsymbol{F}_{l}^{(m)} = \boldsymbol{0}, \quad \forall k \in \{1, \dots, K\}, \\ \forall l \in \{1, \dots, K\} \backslash k.$$
(12)

The DIA method in this case will obtain the solution for the precoders from the conditions

$$\begin{bmatrix} \boldsymbol{H}_{m1} \\ \vdots \\ \boldsymbol{H}_{m,k-1} \\ \boldsymbol{H}_{m,k+1} \\ \vdots \\ \boldsymbol{H}_{m,K} \end{bmatrix} \boldsymbol{F}_{k}^{(m)} = \boldsymbol{V}_{m}, \ \forall k = 1, \dots, K, \ \forall m = 1, \dots, M, \quad (13)$$

with predefined matrices V_1, \ldots, V_M satisfying

$$\boldsymbol{V}_1 + \dots + \boldsymbol{V}_M = \boldsymbol{0}. \tag{14}$$

After the complete cancellation of the interference by the DIA algorithm, the received signal at the MS_k is

$$\boldsymbol{r}_{k} = \left(\sum_{m=1}^{M} \boldsymbol{H}_{mk} \boldsymbol{F}_{k}^{(m)}\right) \boldsymbol{x}_{k} + \boldsymbol{n}_{k}.$$
 (15)

From (15), we conclude that linear receive equalization $\hat{x} = G_k r_k$ to handle the interstream interference is possible if the equivalent channels

$$H_{k}^{eq} = \sum_{m=1}^{M} H_{mk} F_{k}^{(m)}$$
(16)

are estimated by the users (similar to the multiuser MIMO case). The solutions of interest for the receiver are, e.g.,

• ZF equalization

$$\boldsymbol{G}_{k} = \boldsymbol{H}_{k}^{eq+}, \qquad (17)$$

• minimum mean square error (MMSE) equalization $\boldsymbol{G}_{k} = \left(\boldsymbol{H}_{k}^{eqH}\boldsymbol{H}_{k}^{eq} + \sigma^{2}\boldsymbol{I}\right)^{-1}\boldsymbol{H}_{k}^{eqH}.$ (18)

IV. SIMULATION RESULTS

We show the performance results on 3 system configurations, denoted by A1-A2-A3-A4, where A1 is the number of cooperating BSs, A2 is the number of antennas per BS, A3 is the total number of users in the cluster, and A4 is the number of antennas per user. The wireless flat fading channel coefficients are assumed to be i.i.d. Gaussian distributed with zero mean and unit variance. Similarly to [12], we define the system signal-to-noise ratio (SNR) as SNR $\triangleq P_{tot}/\sigma^2$.

In the single-antenna MS case, we compare the DIA and DIA-UPC strategies with the cooperative multicell precoder (CMP) from [11] (using the heuristic power allocation scheme for the



Fig. 4. Average sum rates for the configuration 2-1-2-1.



Fig. 5. Average sum rates for the configuration 2-3-2-1.



Fig. 6. Average sum rates for the configuration 2-2-2-2 with 2 data streams for each MS.

underdetermined case suggested in this paper), which appears to be the best approach from the literature for the scenario of interest [11,12]. We use the joint ZF (JZF) approach from a fully cooperative, network MIMO system [14] (with optimal power allocation by convex optimization on top of a joint ZF precoder) as an achievable DoF indicator for both MISO and MIMO cases.

In Fig. 4, the average sum rates over 1000 random channel realizations are shown for the configuration 2-1-2-1. One can notice that the DIA approach extracts all available DoFs in this case, and that its performance is quite close to the JZF. For the SNR values between 3 and 10 dB, the DIA-UPC, relaxing the total interference cancellation constraint of DIA (which is known to be suboptimal in low SNR regimes), appears to be the best strategy. Therefore, it can be concluded that DIA and DIA-UPC outperform significantly the reference CMP scheme in a very large SNR range. Here, it should be remarked that, apart from the offline agreement on the interference placement, the backhaul burden of the DIA-UPC is exactly the same as in the CMP, while the DIA has additionally one scalar value (common power scaling) that has to be exchanged in a fast manner (cf. Section III).

Fig. 5 shows the performance results for the configuration 2-3-2-1 depicted in Fig. 3. DIA extracts here more DoFs compared to the reference scheme, and yields large gains in the high SNR regime. In the medium SNR range (11-14dB), DIA-UPC has the best performance. For this configuration, in the low SNR regime, CMP remains to be a good choice. This can be explained by the nature of the CMP scheme which converges to an (optimal) MRT solution for low SNR, while DIA benefits from exploiting the available DoFs in the system.

Finally, in Fig. 6, we give results for the full MIMO 2-2-2-2 setup, with each MS receiving 2 independent data streams (a scenario where CMP is not applicable). It can be seen that DIA achieves the maximum number of DoFs also in this configuration (cf. the curves' slopes in the high SNR regime), and that its loss w.r.t. the JZF strategy is relatively small. Regarding the receiver, as expected, the MMSE equalization outperforms ZF (and even JZF) in the low SNR regime.

At this point, it should be mentioned that the vectors \boldsymbol{v}_m and matrices \boldsymbol{V}_m from (6) and (14) are selected in the numerical examples given above as $e^{i2\pi m/M}[1 \ 0 \dots 0]^T$ and $e^{i2\pi m/M}[I \ 0 \dots 0]^T$, respectively. This is a good choice particularly for the DIA-UPC strategy, as its suboptimal scaling of the precoders will not affect the interference components which are forced to be zero, so the number of these components should be maximized. Further optimization of the offline agreed vectors (matrices) \boldsymbol{v}_m (\boldsymbol{V}_m) remains, however, as an interesting topic for the future work.

V. CONCLUSIONS

It is shown how in certain scenarios of practical interest only with local CSI and data sharing it is possible to extract a large part of the rate gain that multi-cell cooperation with full information exchange provides. In other words, the results imply that high latency of the backhaul does not necessarily prevent the benefits of multi-cell cooperation. The plans for the future work include further improvement of performance in the low SNR regime, optimization of the interference placement and factors multiplying the desired signals, analysis of synchronization errors, and more sophisticated distributed power control.

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References

- M. K. Karakayali, G. J. Foschini, and R. A. Valenzuela, "Network coordination for spectrally efficient communications in cellular systems," *IEEE Trans. Wireless Communications*, vol. 13, no. 4, pp. 56-61, Aug. 2006.
- [2] P. Marsch and G. P. Fettweis, Cooperative Multi-Point in Mobile Communications: From Theory to Practice, Cambridge University Press, 2011.
- [3] H. Dahrouj and W. Yu, "Coordinated beamforming for the multicell multi-antenna wireless system," *IEEE Trans. Wireless Comm.*, vol. 9, no. 5, pp. 1748-1759, May 2010.
- [4] 3GPP Technical Specification Group Radio Access Network, "Coordinated multi-point operation for LTE physical layer aspects (Release 11)," 3GPP TR 36.819, v11.1.0, Dec. 2011.
- [5] IEEE 802.3 Standardization Group. Amendment 4: Media Access Control Parameters, Physical Layers and Management Parameters for 40 Gb/s and 100 Gb/s Operation, Jun. 2010.
- [6] N. Cvijetic, A. Tanaka, M. Cvijetic, Y.-K. Huang, E. Ip, Y. Shao, and T. Wang, "Novel optical access and digital processing architectures for future mobile backhaul," *IEEE Journal of Lightwave Technology*, vol. 31, no. 4, pp. 621-627, Feb. 2013.
- [7] N. Golrezaei, A. F. Molisch, A. G. Dimakis, and G. Caire, "Femtocaching and device-to-device collaboration: A new architecture for wireless video distribution," *IEEE Comm. Magazine*, vol. 51, no. 4, pp. 142-149, Apr. 2013.
- [8] A. Liu and V. K. N. Lau, "Mixed-timescale precoding and cache control in cached MIMO interference network," *IEEE Trans. on Signal Processing*, vol. 61, no. 24, pp. 6320-6332, Dec. 2013.
- [9] L. Thiele, M. Olbrich, M. Kurras, and B. Matthiesen, "Channel aging effects in CoMP transmission: Gains from linear channel prediction", in *Proc. Asilomar 2011*, Pacific Grove, CA, 2011.
- [10] T. Gou and S. A. Jafar, "Optimal use of current and outdated channel state information – Degrees of freedom of the MISO BC with mixed CSIT," *IEEE Comm. Letters*, vol. 16, no. 7, pp. 1084–1087, Jul. 2012.
- [11] E. Björnson, R. Zakhour, D. Gesbert, and B. Ottersten, "Cooperative multicell precoding: Rate region characterization and distributed strategies with instantaneous and statistical CSI," *IEEE Trans. on Signal Processing*, vol. 58, no. 8, pp. 4298-4310, Aug. 2010.
- [12] R. Zakhour and G. Gesbert, "Distributed multicell-MISO precoding using the layered virtual SINR framework," *IEEE Trans. on Wireless Communications*, vol. 9, no. 8, pp. 2444-2448, Aug. 2010.
- [13] E. Björnson and E. Jorswieck, "Optimal Resource Allocation in Coordinated Multi-Cell Systems", *Foundations and Trends in Comm.* and Inf. Theory, vol. 9, no. 2-3, pp. 113-381, 2013.
- [14] F. Boccardi and H. Huang, "Optimum power allocation for the MIMO BC zero-forcing precoder with per-antenna power constraints," in *Proc.* 40th Annual Conf. Inf. Sciences Syst., Mar. 2006.
- [15] R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge University Press, 1985.
- [16] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Trans. Veh. Technol.*, vol. 53, no. 1, pp. 18–28, 2004.
- [17] S. A. Jafar, "Interference Alignment A New Look at Signal Dimensions in a Communication Network," *Foundations and Trends in Comm. and Inf. Theory*, vol. 7, no. 1 pp. 1-134, 2011.