DERIVATION OF ANALYTICAL EXPRESSIONS FOR FLEXIBLE PR LOW COMPLEXITY FBMC SYSTEMS

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ABSTRACT

Compared to OFDM, Filter Bank Multi-Carrier (FBMC) systems can offer improved Time-Frequency Localization (TFL) features. In this paper, using the TFL criterion, we present an analytical derivation of FBMC prototype filters that insures the Perfect Reconstruction (PR) property for the two most important classes of FBMC systems: Filtered MultiTone (FMT) and OFDM with Offset QAM.

Index Terms— FBMC; FMT; OFDM; OFDM/OQAM; Time-frequency localization.

1. INTRODUCTION

Since its adoption for digital audio and video broadcasting (DVB, DAB), the orthogonal frequency division multiplexing (OFDM) has gained a growing interest for transmission over multipath channels and is undoubtedly now the flagship of all MC modulations [1]. However, researchers still expect to find better transmission schemes and most often propose alternatives that are based on FBMC schemes which can also take advantage of efficient implementation algorithms using the discrete Fourier Transform [2]. In these FBMC schemes, the rectangular OFDM window is replaced by a smoother one, thus leading to an improved frequency behavior.

To modify the window function, while still preserving the PR property, one has to find Degree of Freedom (DoF). For FMT, which basic scheme is reported in Fig. 1, the DoF comes from the oversampling ratio N/M, with $N > M$. In the case of the OFDM/OQAM system, the DoF is obtained by a decomposition of the input QAM data symbols, denoted $c_{m,n}$ in Fig. 1, taking alternately in time (index n) and frequency (index m), their real and imaginary part [3, 4].

For these two types of modulated systems, each constituent filter of the filter bank is obtained by an exponential modulation of the window function, the "so-called" prototype filter, we denote by p. Taking, as an example, the M-carrier FMT scheme depicted in Fig. 1, the time response for the M filters, $F_m(z), m = 0, \cdots M-1$, of the synthesis filter bank can be expressed as $f_m[k] = p[k]\exp(j\frac{2\pi mk}{M})$, with $j^2 = -1$ and $k = 0, \cdots, L-1$, for a prototype filter of length L. Thus, there is only one unique filter to determine, since all subchannels filters are derived from p. Indeed, as we assume the considered FBMC systems are orthogonal, the $H_m(z), m = 0, \cdots M-1$ filters are also derived from p.

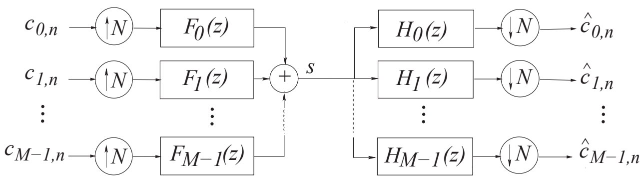

Fig. 1. Schematic representation of a FMT system ($N > M$).

The design problem is to find the L variables $p[k]$ that lead to an appropriate waveform, s, for a given transmission channel. Generally, the solution results from an optimization procedure carried out for a given design criterion, for instance minimization of the out-of-band energy [5, 4, 6] or maximization of the time-frequency localization (TFL) [3], [7]. In previous papers, the authors only focus on the design of a single type of FBMC system, e.g. FBMC/FMT in [8], [5], [9] or FBMC/OQAM in [3, 4]. Generally also, it is considered that the length of the prototype filter must be much higher than the number of subchannels, e.g. $L = mM$ with $m = 3$ or 4 in [4] or $m \in \{18, 20, 30\}$ in [5]. However, it has been shown recently that, for time-varying transmission channels, shorter prototype filters ($L = N$) could also provide good performances with FBMC/FMT [6, 10] or FBMC/OQAM systems [11]. This is particularly true for prototype filters that are designed according to the TFL criterion.

Therefore, in this paper, we only focus on the TFL criterion, for which we derive closed-form expressions of the prototype filter coefficients. Our approach may lead to flexible implementation for applications where the system parameters, e.g. the number of subchannels, may vary in time.

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Compared to [9], our derivation now also includes the case of FBMC/OQAM systems. We also provide an accuracy measure which gives a precise indication of how close we are from TFL optimality. This measure shows that we can now insure a nearly optimal result for both FBMC (FMT and OQAM) systems containing up to thousands of subchannels.

Our paper is organized as follows. In Section 2, we review the main features of the low complexity PR-FBMC systems we are considering and also present the TFL optimization problem. Section 3 presents an analysis of the optimized prototype filters. Our analytical derivations are detailed in Section 4 and design examples are provided in Section 5.

2. GENERAL PRESENTATION

2.1. Low complexity FBMC systems

The digital baseband of the FBMC systems we consider in this paper can be represented by the generic modulator structure shown in Fig. 2. As the implementation of the prototype filter only involves one multiplication per sub-carrier, all the derived FBMC systems, detailed below, are of relatively low complexity.

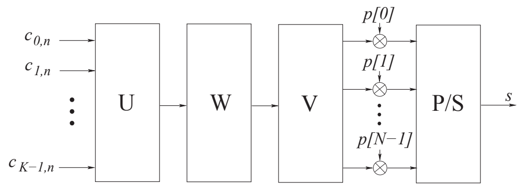

Fig. 2. Generic scheme for low complexity FBMC systems.

2.1.1. The CP-OFDM reference

For the CP-OFDM modulator, we have M data symbol inputs, i.e. $K = M$. U, W and V correspond to the $M \times M$ identity matrix, the M-size discrete Fourier transform (DFT) and the CP extension, respectively. Otherwise said, the CP-length is $N - M$.The $p[k]$ coefficients are the constant value numbers of the rectangular window.

2.1.2. FBMC/FMT

Then, we also have $K = M$, while U and W are the M-size identity and DFT matrices, respectively. V corresponds to a $N \times M$ cyclic extension matrix [10].

2.1.3. FBMC/OQAM

Then, we have $K = N = 2M$. The U block corresponds to the conversion process transforming the QAM input complex symbols into OQAM real symbols [3], [7]. W and V are N-size DFT and identity matrices, respectively.

Though, the two FBMC systems have different features they may lead, for $N = 2M$, to an identical filter design problem for finding the $p[n]$'s.

2.2. The PR FBMC condition

If the $p[k]$ coefficients in Fig. 2 satisfy orthogonality conditions then we get a PR system, i.e. $\hat{c}_{m,n} = \alpha c_{m,n}$, $m = 0 \cdots K-1, \forall n$, with $\alpha > 0$. Based on [12, (10)], the PR condition for the prototype filter of FMT systems may be written, for $0 \le k \le M-1$, as

$$\sum_{\nu} p[k+\nu M]\, p[k+\nu M+sN] = \delta_s,\ s \ge 0, \qquad (1)$$

where $p[n] = 0$ for $n < 0$ or $n \ge L$ and $\delta_s = 1$ if $s = 0$ and 0 if $s \neq 0$. Then, setting $N = 2M$, we exactly recover the PR condition given in [13] for an M-band cosine modulated filter banks (CMFB) using a linear phase prototype filter, which is also the one of a N-sub-carrier OFDM/OQAM system [3]. Therefore, even if the corresponding realization schemes are different, a prototype filter satisfying (1) can be used for the two main classes of FBMC systems. In what follows, we only focus on (1) in the case where $L = N$.

Let M_0 and N_0 be relatively prime integers with $1 \le M_0 < N_0$. For any integer $\Delta \ge 1$, let $M = \Delta M_0$, $N = \Delta N_0$, $L = \Delta N_0$. To get a maximum theoretical spectral efficiency we impose that $N_0 = M_0 + 1$. The unique solution can be expressed using a set of Δ angles [6, (78)]

$$\begin{aligned} &p[i] = \cos\theta_i,\ \ p[M+i] = \sin\theta_i,\ 0 \le i \le \Delta - 1, \\ &p[k] = 1,\ \Delta \le k \le M-1. \end{aligned} \qquad (2)$$

The problem is to optimize this set of Δ angles.

2.3. Optimization variants

Based on [14] and denoting $||P|| = \left(\sum_n p[n]^2\right)^{\frac{1}{2}}$, the TFL measure, ξ, for the discrete sequence $\{p[k]\}$ is such that

$$\begin{aligned} T &= \frac{\sum_n (n-\frac{1}{2})\left(p[n]+p[n-1]\right)^2}{\sum_n \left(p[n]+p[n-1]\right)^2}, \\ m_2 &= \frac{1}{4||P||^2}\sum_n (n-\frac{1}{2}-T)^2\left(p[n]+p[n-1]\right)^2, \\ M_2 &= \frac{1}{||P||^2}\sum_n \left(p[n]-p[n-1]\right)^2, \\ \xi &= \frac{1}{\sqrt{4m_2M_2}}. \end{aligned} \qquad (3)$$

Then, the optimal TFL is the one satisfying $\max_{\{p[k]\}} \xi(p)$ or $\max_{\theta_i} \xi(p)$. If instead, we take advantage of the compact representation (CR) method [15], the optimization is carried out using a parameter set with reduced dimension. The CR idea is to express the θ_i angles as follows

$$\theta_i = f(x_i) \text{ where } x_i = \frac{2i+1}{2\Delta} \qquad (4)$$

where f is, in general, a polynomial function and the coefficients of the CR are its coefficients. As the degree of f can be much smaller than Δ, while still providing a nearly optimal solution, the corresponding design time can be significantly reduced compared to the one resulting from the two direct formulations. Furthermore, as shown in [9], reducing f to a simple linear function allows us to get very simple closed-form expressions for the $p[n]'s$. Compared to [9], our aim is to consider a wider range for the Δ parameter, i.e. a higher number of subchannels. In order to also provide a design solution for FBMC/OQAM systems, we also include the case where $M_0 = 1$, i.e. $N = 2M$ (see subsection 2.1.3).

3. ANALYSIS OF PR-FBMC SYSTEMS

3.1. Phase linearity

The Δ-order polyphase decomposition of the prototype filter $P(z)$, z-transform of the $\{p[n]\}$ sequence, reads as

$$P(z) = \sum_{i=0}^{\Delta-1} z^{-i} P_i(z^{\Delta}), \tag{5}$$

where, using (2), we get $P_i(z) = \sum_{n=0}^{M_0} p_i[n] z^{-n}$ with $p_i[0] = \cos\theta_i$, $p_i[M_0] = \sin\theta_i$ and $p_i[n] = 1$, $1 \leq n < M_0$. An optimization of the Δ independent angles leads to the results in Fig. 3. The corresponding graphs, for $M_0 = 1, 2, 3, 5$ and $\Delta = 25$, show the variations of $\psi_{M_0,\Delta,i} = \theta_{M_0,\Delta,i} - \frac{\pi}{2}(1 - \frac{2i+1}{2\Delta})$, where $\theta_{M_0,\Delta,i}$ designates the value of the i-th optimal angle.

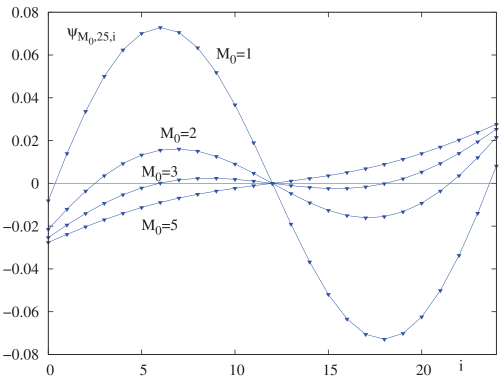

Fig. 3. Variations of the optimal angles for $M_0 = 1, 2, 3, 5$ and $\Delta = 25$.

Besides, the regular behavior of the angles $\theta_{M_0,\Delta,i}$ w.r.t. i, already observed in [15], one can also see that an approximate symmetry property appears with $\psi_{M_0,\Delta,i} \cong -\psi_{M_0,\Delta,\Delta-1-i}$, i.e. $\theta_{M_0,\Delta,i} \cong \frac{\pi}{2} - \theta_{M_0,\Delta,\Delta-1-i}$. This observation, also used in [9], indicates that optimal $P(z)$ have a nearly linear phase behavior, i.e. $p[k] \cong p[N-1-k]$. Thus, in order to reduce the number of parameters, we choose to use a CR, we denote by f, that will structurally guarantee that $P(z)$ is symmetrical, expressing it as follows

$$f(x) = \frac{\pi}{4} + t \sum_{k=0}^{d-1} \beta_k T_{2k}(t), \; t = 2x - 1, \tag{6}$$

where $T_n(t)$ is the Chebyshev polynomial of degree n. This function f depends on the d parameters β_k, $0 \leq k \leq d-1$ where d designates the degree of the CR. Note that f satisfies the property symmetry $f(1-x) = \frac{\pi}{2} - f(x)$ that corresponds to the one characterizing the optimal angles.

3.2. Performance w.r.t. the TFL criterion

To validate the choice of this new CR (6), we compare the results it provides with the ones where the angles are directly optimized. Optimization of the TFL criterion is carried out for d values going from 1 to 4 and for $1 \leq \Delta \leq 200$.

When a direct optimization of the angles leads to the optimal TFL denoted $\xi_{\text{opt}}(M_0, \Delta)$, we compute the relative error w.r.t. $\xi_{\text{opt}}^{(d)}(M_0, \Delta)$, optimal localization for the degree d, by

$$\varepsilon^{(d)}(M_0, \Delta) = \sigma \log_{10} \left| \frac{\xi_{\text{opt}}^{(d)}(M_0, \Delta) - \xi_{\text{opt}}(M_0, \Delta)}{\xi_{\text{opt}}(M_0, \Delta)} \right|, \tag{7}$$

where $\sigma = -1$ if the value $\xi_{\text{opt}}(M_0, \Delta)$ is approached from below, and $+1$ otherwise. In this later case, the CR gives a better result than the optimization over angles.

Using this measure, a large set of optimizations leads to several observations. For a sufficient value of Δ, when $M_0 = 1$ or 2 a degree $d = 3$ is required to get $\varepsilon^{(d)}(M_0, \Delta) \leq 10^{-4}$, while for higher values of M_0, say e.g. $M_0 = 10$, $d = 2$ is enough and then when $d = 4$, we get $\xi_{\text{opt}}^{(4)}(M_0, \Delta) > \xi_{\text{opt}}(M_0, \Delta)$, i.e. $\sigma = +1$.

As we target an accuracy of 10^{-4} in a large range of Δ values, $\Delta \in [4, 200]$, according to the value of M_0, we have to use a CR with either $d = 2$ or 3. Thus, for $M_0 \in [4, 20]$, we will use a database containing the corresponding optimal values found with (6) for $\beta_0(M_0, \Delta)$ and $\beta_1(M_0, \Delta)$. While for $M_0 = 1, 2$ or 3, the corresponding database will include furthermore the optimal values found for the three β_k parameters, i.e. including the set of optimized $\beta_2(M_0, \Delta)$ values.

4. ANALYTICAL DERIVATION OF PR FBMC

4.1. Compact representation with $d = 2$

Setting

$$\gamma_0(M_0, \Delta) = \frac{\pi}{4} + \beta_0(M_0, \Delta) + \beta_1(M_0, \Delta), \tag{8}$$

and $t = 2x - 1$ the CR in (6) writes for $0 \leq x \leq 1$, as

$$f(x) = \frac{\pi}{2}(1 - x) + t[\gamma_0(M_0, \Delta) + 2(t^2 - 1)\beta_1(M_0, \Delta)]. \tag{9}$$

In a first step, we need to study the behavior of $\beta_1(M_0, \Delta)$ and $\gamma_0(M_0, \Delta)$, as a function of Δ and M_0. Observing firstly that with our database $\beta_1(M_0,\Delta)$ approximatively behaves as a function given by $\beta_1(M_0,\Delta) = a(M_0) + b(M_0)/\Delta$, a linear regression gives the coefficients $a(M_0)$ and $b(M_0)$ which minimize

$$\sum_{\Delta=4}^{200}\left[\beta_1(M_0,\Delta) - \left(a(M_0) + \frac{b(M_0)}{\Delta}\right)\right]^2. \quad (10)$$

In Step 2, we observe that $1/\sqrt{a(M_0)}$ and $1/\sqrt{b(M_0)}$ behave as nearly linear functions of M_0. Again, a linear regression can be used to find them, leading to

$$a(M_0) = \frac{X_0}{(X_1 + M_0)^2},\ b(M_0) = \frac{X_2}{(X_3 + M_0)^2}, \quad (11)$$

where each $X_i, i = 1,\ldots,4$, denotes a constant term.

In Step 3, it can be noticed that, for fixed M_0, the inverse of $\gamma_0(M_0,\Delta)$ has a nearly linear variation in function of Δ. Two functions, denoted $c(M_0)$ and $d(M_0)$, can be then determined by linear regression

$$\frac{1}{\gamma_0(M_0,\Delta)} \sim c(M_0) + d(M_0)\Delta. \quad (12)$$

Then, setting $X_4 = 1.273$, value obtained after a few trials of a dichotomous process, we are able to get a function $(c(M_0) - X_4)^{-1/2}$ which is nearly linear. Similarly, setting $X_7 = 1.273$, we get another linear function $(d(M_0) - X_7)^{-1/2}$ w.r.t. M_0.

By linear regression, we find the equations of the straight lines approximating these curves

$$c(M_0) = X_4 + \frac{X_5}{(M_0+X_6)^2},\ d(M_0) = X_7 + \frac{X_8}{(M_0+X_9)^2}, \quad (13)$$

where each $X_i, i = 4,\ldots,9$, denotes a constant term.

Using (11) and (13) with the constant terms $X = (X_i, i = 0,\ldots,9)$, $\beta_1(M_0,\Delta)$ and $\gamma_0(M_0,\Delta)$ functions are thus approximated as follows

$$\tilde{\beta}_1(M_0,\Delta,X) = a(M_0) + \frac{b(M_0)}{\Delta}, \quad (14)$$

$$\tilde{\gamma}_0(M_0,\Delta,X) = [c(M_0) + d(M_0)\Delta]^{-1}. \quad (15)$$

Finally, starting from the initial previously computed values of X, a global optimization is carried out, for $M_0 = 4,\ldots,20$ and $\Delta = 4,\ldots,200$. The aim is to minimize the mean square of $\beta_1(M_0,\Delta) - \tilde{\beta}_1(M_0,\Delta,X)$ and of $\gamma_0(M_0,\Delta) - \tilde{\gamma}_0(M_0,\Delta,X)$. The optimized values of $X_i, i = 0,\ldots,9$ are given in Table 1.

To summarize, starting from (9) and using (12) for γ_0 expression and (10) for β_1 approximation, the Δ angles θ_i with $i = 0,\cdots,\Delta-1$ are computed as follows

$$\theta_i = f(x_i) = \frac{\pi}{2}(1-x_i) + (2x_i - 1)\left[\frac{1}{c(M_0) + d(M_0)\Delta} + 2[(2x_i-1)^2 - 1][a(M_0) + \frac{b(M_0)}{\Delta}]\right], \quad (16)$$

i	X_i	i	X_i
0	0.19403124832632	5	2.90492587969539
1	0.40864162382945	6	0.86264166373416
2	0.35329881606485	7	1.27240200581068
3	0.39920459787503	8	0.51760963875876
4	1.27060234434206	9	0.52820298059447

Table 1. Optimized constant terms $X_i,\ 0 \le i \le 9$ in the approximations of $\beta_1(M_0,\Delta)$ and $\gamma_0(M_0,\Delta)$.

where $x_i = \frac{2i+1}{2\Delta}$.

4.2. Compact representation with $d = 3$

For $M_0 \in \{1,2,3\},\ 4 \le \Delta \le 200$, a degree $d = 3$ is required, so we now set $\gamma_0(M_0,\Delta) = \frac{\pi}{4} + \beta_0(M_0,\Delta) + \beta_1(M_0,\Delta) + \beta_2(M_0,\Delta)$, and then the CR in (6) writes as

$$f(x) = \frac{\pi}{2}(1-x) + \gamma_0(M_0,\Delta)t + 2t(t^2-1)[\beta_1(M_0,\Delta) + 4\beta_2(M_0,\Delta)t^2],\ t = 2x-1,\ 0 \le x \le 1, \quad (17)$$

Again, we proceed to a succession of linear regressions and to a final global optimization that lead to the following set of relations, where constants $X_{M_0,i}$ are reported in Table 2,

$$\gamma_0(M_0,\Delta) \sim \frac{1}{X_{M_0,0} + X_{M_0,1}\Delta}, \quad (18)$$

$$\beta_1(M_0,\Delta) \sim X_{M_0,2} + \frac{1}{X_{M_0,3} + X_{M_0,4}\Delta}, \quad (19)$$

$$\beta_2(M_0,\Delta) \sim X_{M_0,5} + \frac{1}{X_{M_0,6} + X_{M_0,7}\Delta}. \quad (20)$$

i	$X_{1,i}$	i	$X_{1,i}$
0	4.1284847578	4	$-2.1107642825\ 10^1$
1	1.9727736832	5	$-6.6774831778\ 10^{-3}$
2	$1.2781855004\ 10^{-1}$	6	$-1.0150558822\ 10^2$
3	$-1.4505800309\ 10^2$	7	$1.9143799092\ 10^{-2}$
i	$X_{2,i}$	i	$X_{2,i}$
0	1.8972250436	4	$-9.2112632592\ 10^1$
1	1.4476020206	5	$-5.2062788263\ 10^{-3}$
2	$4.2968806649\ 10^{-2}$	6	$5.9290534083\ 10^2$
3	$-7.7723347312\ 10^2$	7	$9.5812941281\ 10^1$
i	$X_{3,i}$	i	$X_{3,i}$
0	1.5475698371	4	$-2.9101929435\ 10^2$
1	1.3525325059	5	$-2.4560808315\ 10^{-3}$
2	$2.0804395123\ 10^{-2}$	6	$3.6665827460\ 10^2$
3	$-4.5492785604\ 10^3$	7	$2.2289866975\ 10^2$

Table 2. Optimized constant terms $X_{M_0,i}$ for $M_0 = 1,2,3$, $0 \le i \le 7$.

5. DESIGN EXAMPLES

Our analytical derivations guarantee that $\xi_{\text{opt}}^{(d)} < 10^{-4}$ and, furthermore, give good results outside the ranges of optimization. In Figs. 2 and 3, which show the time and frequency representations of prototype filters obtained for $M_0 = 1, 8, 32$ and $\Delta = 2048$, the relative errors for optimized TFL are equal to $5.7\ 10^{-4}, 4.37\ 10^{-6}$ and $1.54\ 10^{-8}$, respectively. If, as expected, the TFL decreases with increasing M_0: $\xi = 0.906$ ($M_0 = 1$); 0.389 ($M_0 = 8$); 0.195 ($M_0 = 32$), these values outperform the ones we get for OFDM: $\xi = 0.019$; $9.02\ 10^{-3}$; and $4.71\ 10^{-3}$, respectively.

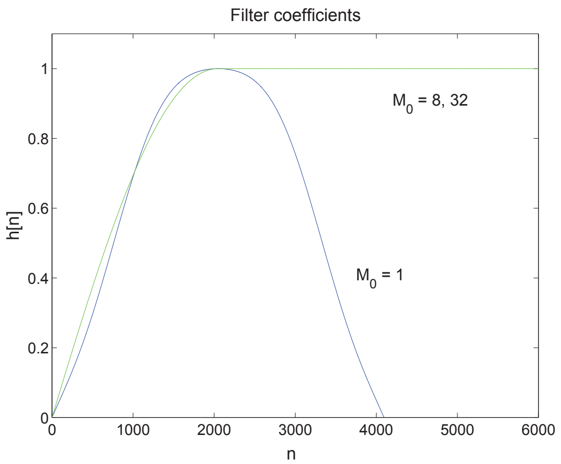

Fig. 2. Time responses of the optimal TFL prototype filters for $\Delta = 2048$ and $M_0 = 1, 8, 32$. For $M_0 = 8$ and 32 only the 6000 first coefficients are plotted.

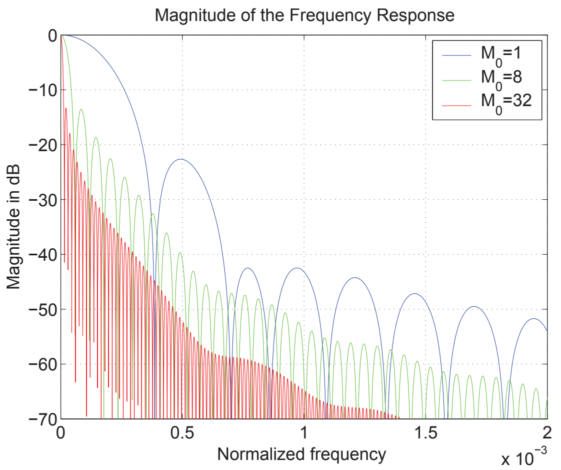

Fig. 3. Magnitude frequency responses of the optimal TFL prototype filters for $\Delta = 2048$ and $M_0 = 1, 8$ and 32..

6. CONCLUSION

Our analysis of low complexity FBMC systems has confirmed that optimal prototype filters w.r.t. TFL are linear phase [9]. Then, using an appropriate compact representation of low degree, we could analytically derive, for a broad range of FMBC parameters, accurate values of the prototype filter coefficients.

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